Estimating the Parameters of a Duffing Equation in which the Time-Series Data of the Solution Becomes Chaotic

Kimihiko UENO† and Yoshinobu HIDA ‡
† Tokyo University of Marine Science and Technology
5-7, Konan 4-cyome, Minatoku, Tokyo 108-8477, Japan
‡ Graduate school of Marine Science and Technology Tokyo University of Marine Science and Technology
5-7, Konan 4-cyome, Minatoku, Tokyo 108-8477, Japan

Abstract

One of the characteristics of chaotic systems is their sensitivity to the initial conditions. As a result, time-series data that become chaotic can be predicted over the short term, but not over the long term. This research considers the effect of errors in the parameters of a chaotic system. A genetic algorithm was used to estimate these parameters, and the resulting error in time-series predictions given by the chaotic system was examined.

1 Introduction

One of the well-known characteristics of chaotic systems is their sensitive dependency to initial conditions [1]. As a result, it is possible to make predictions over the short term, but not over the long term for chaotic time-series data. In contrast, there are few studies that show the effects of parameter estimation errors in equations representing chaotic systems, on the resulting prediction errors in chaotic time-series. The Duffing equation may be used to express the equation of motion with a single degree of freedom, if the nonlinearity of the restoring term cannot be ignored. Ueda [4, 5, 6] showed that the time-series solution given by the Duffing equation becomes chaotic depending on the parameters taken. In this study, we attempted to estimate parameters by using the genetic algorithm proposed by Ueno et al.[7], in cases where the time-series solutions given by the Duffing equation become chaotic. In addition, we numerically solved the Duffing equation possessing the estimated parameters, made predictions, and considered the effects of parameter estimation errors on prediction error.

2 Data Used for Parameter Estimation

The first data used for parameter estimation was the numerical solution of the following differential equation, obtained by the Runge-Kutta fourth-order method. The other is data with added noise, as described later. In the Runge-Kutta method, the time interval was set to 1.0×10^{-5}[sec] when calculating the Poincaré section, and 1.0×10^{-2} [sec] for all other cases[3]. We assumed the initial condition, $\phi(0) =$

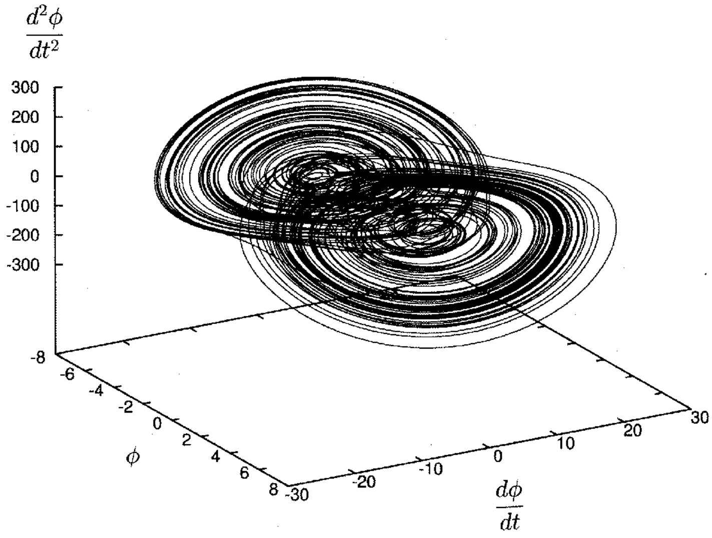

Fig. 1: Phase diagram of Ueda model.

0, $\frac{d\phi}{dt}|_{t=0} = 0$. We referenced the study by Kan and Taguchi [2] for parameter settings. For the rest of this study, we refer to the model represented by the following differential equation as the 'Ueda model.' Fig.1 shows the phase diagram of the Ueda model. Fig.2 shows the Poincaré section of the Ueda model.

$$\frac{d^2\phi}{dt^2} + b_1\frac{d\phi}{dt} + c_1\phi + c_3\phi^3 = P\cos\omega t \tag{2.1}$$
$$b_1 = 0.1, c_1 = 1, c_3 = 1, P = 60, \omega = 2$$

As described above, we used two sets of data for parameter estimation. The first does not contain noise (See Fig.3). We hereinafter refer to this data as Data-1, or true value. The second is the true value (Data-1) with added white noise (See Fig.4). We hereinafter refer to this data containing noise as Data-2. Fig.5 shows the time-series data of this added white noise. Fig.6 shows the power spectral density function of this white noise. We used the genetic algorithm to carry out parameter estimations from the data in the 0-20 [sec] interval, for Data-1 and Data-2 (See Figs. 3 and 4)[7].

3 Results and Analysis

3.1 Estimation of Parameters

Table 1 shows the parameters estimated from time-series data not containing noise (Data-1), while Table 2 shows the parameters estimated from time-series data containing noise (Data-2). Based on the estimated parameters, we numerically solved the differential equation of the Ueda model; Fig.7 and Fig.8 show the calculated results compared to the true values. Fig.7 shows the resulting values when using parameters estimated from Data-1 (See Table 1), while Fig.8 shows those resulting from parameters estimated from Data-2 (See Table 2).

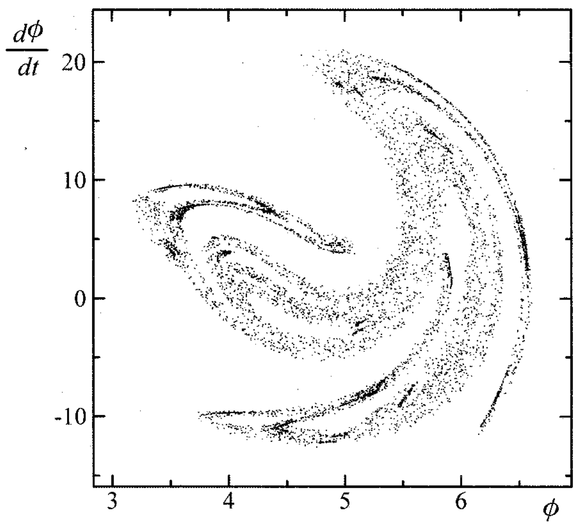

Fig. 2: Poincaré section of Ueda model.

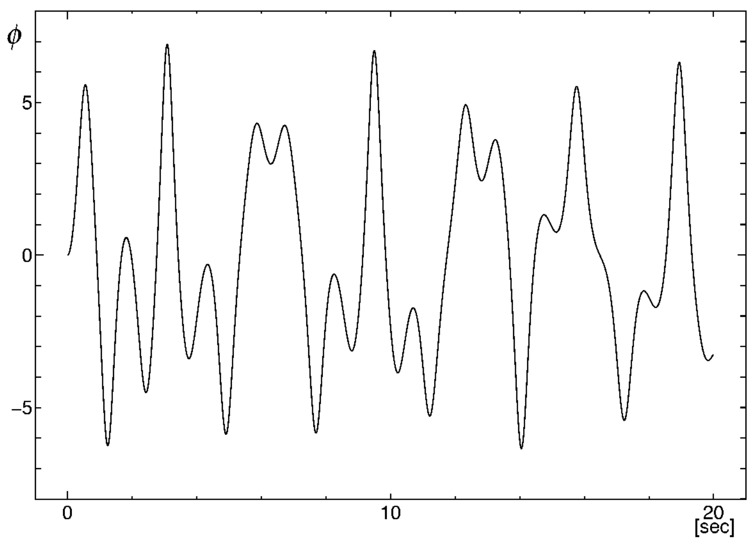

Fig. 3: True value of time-series data not containing noise.

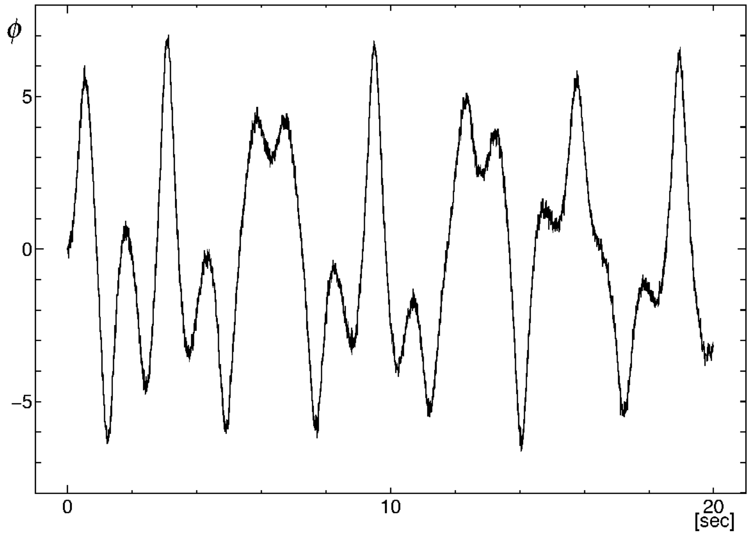

Fig. 4: Time-series data containing noise.

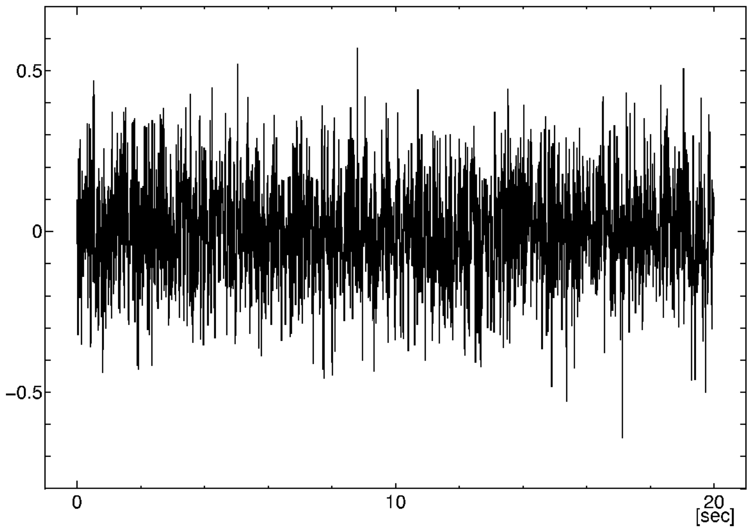

Fig. 5: Time series data of the white noise.

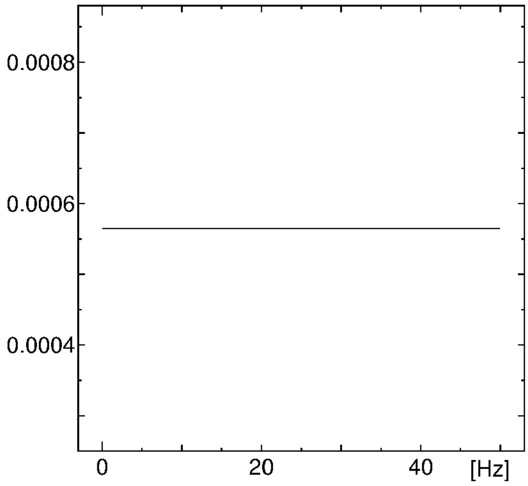

Fig. 6: The power spectral density function of the white noise.

We calculated the RMSE (Root Mean Square Error) using the following equation:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\hat{\phi}_i - \phi_i)^2} \tag{3.1}$$

Here, ϕ_i is the true value and $\hat{\phi}_i$ stands for the estimated value. The parameters and resulting numerical solutions are similar to the true values in the 0-20 [sec] interval used in this estimation, for both of the calculated results estimated from Data-1 and from Data-2.

A large error is seen near $t = 18$ [sec] compared to other areas when the parameters are estimated from Data-2. See the circled area in Fig.8. This is because, in the true value, $b_1 = 0.1$, while $b_1 = 0.10154879$ in the estimated value, causing a slight overestimation. Fig. 9 shows the calculated results, when we set $b_1 = 0.101$ and maintain all other parameter values in Table (apart from b_1) the same. We can confirm that the error becomes small.

Table 1: Parameter estimates from time-series data not containing noise.

Parameter	True	Estimated
b_1	0.1	0.09997711
c_1	1.0	1.00004578
c_3	1.0	1.00120546
P	60	59.96032654
ω	2	2.00000000

3.2 Prediction

Fig.10 and Fig.11 show the values predicted up to 60 [sec] using the parameters estimated from the 0-20 [sec] interval. Fig.10 shows the case where the utilized

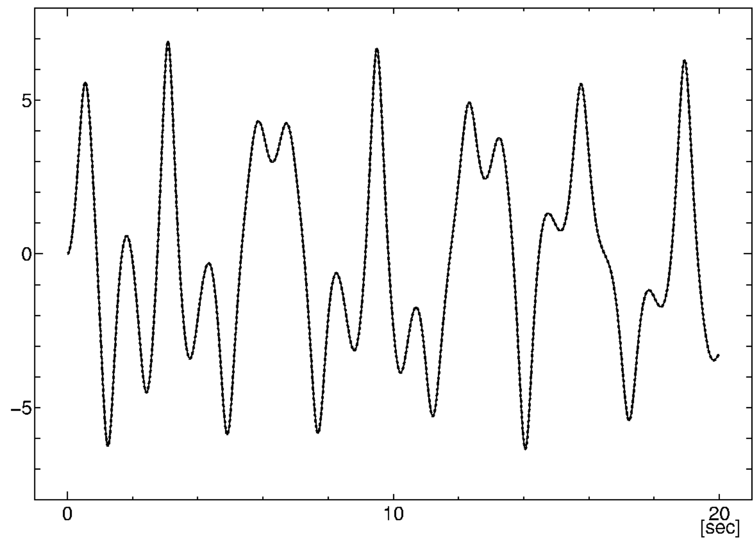

Fig. 7: Comparison of the true and estimated values 1. The case that parameters are estimated from time-series data not containing noise. The dotted line corresponds to the true value and the solid line corresponds to the estimated value. RMSE=0.00203338.

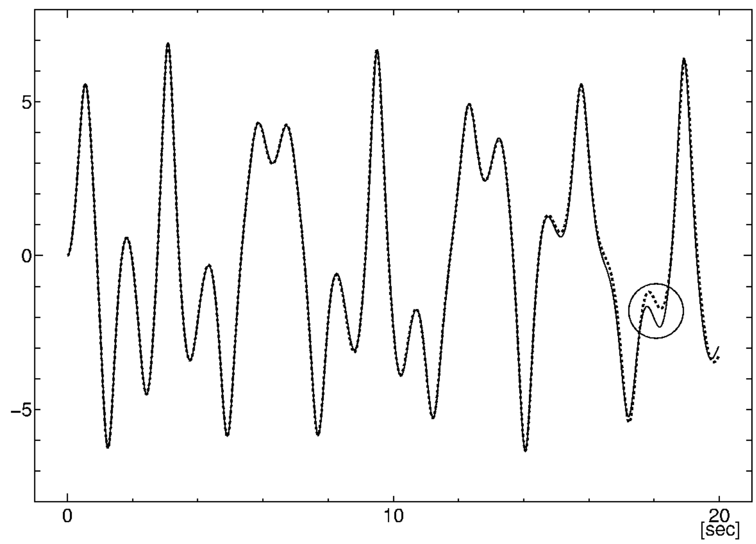

Fig. 8: Comparison of the true and estimated values 2. The case that parameters are estimated from time-series data containing noise. The dotted line corresponds to the true value and the solid line corresponds to the estimated value. RMSE=0.16167712

Table 2: Parameter estimates from time-series data containing noise.

Parameter	True	Estimated
b_1	0.1	0.10154879
c_1	1.0	1.00001526
c_3	1.0	0.99949645
P	60	59.96795605
ω	2	1.99984741

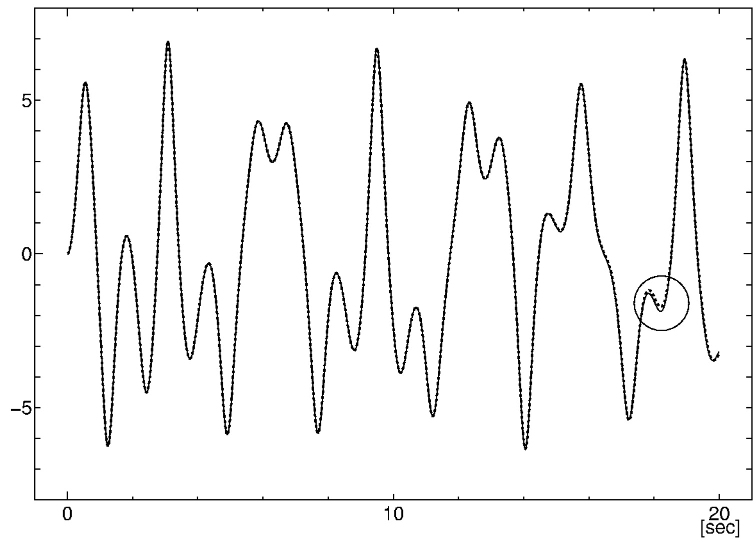

Fig. 9: Comparison of the true and estimated values 3. The dotted line corresponds to the true value and the solid line corresponds to the estimated value. RMSE=0.03537717

parameters were estimated from Data-1, and Fig.11 shows the case where they were estimated from Data-2. Due to the aforementioned overestimation in the attenuation parameter b_1, the prediction error becomes larger at an earlier stage for the results using parameters derived from Data-2 than those derived from Data-1.

4 Conclusion

In the 0-20 [sec] interval used in parameter estimation, both the parameters estimated from Data-1 and Data-2 were close to the true values. In addition, the numerical solutions based on the estimated parameters were also similar to the true values. Thus, the parameter estimation method using a genetic algorithm was also effective for the Duffing equation, where the time-series data, given as solutions, are chaotic. However, it is necessary to sufficiently examine prediction errors caused by parameter estimation errors when making predictions based on the parameters estimated from short intervals.

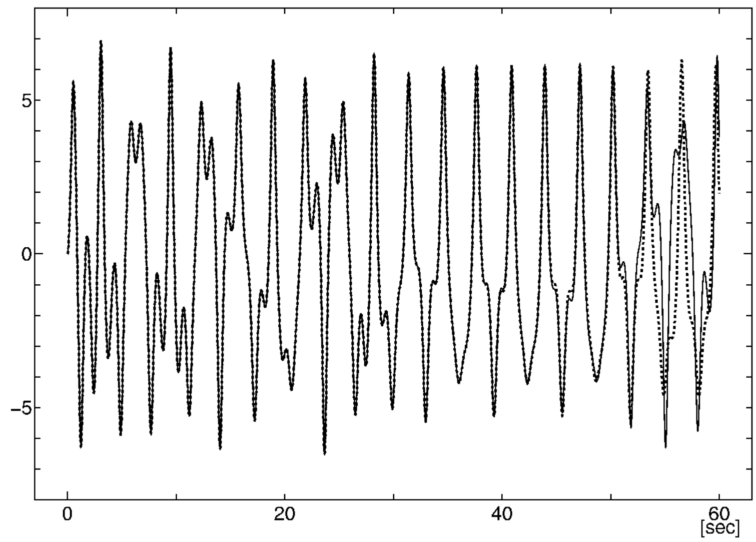

Fig. 10: Comparison of the true and predicted values 1. The case that parameters are estimated from time-series data not containing noise. The dotted line corresponds to the true value and the solid line corresponds to the predicted value. RMSE=0.85898294

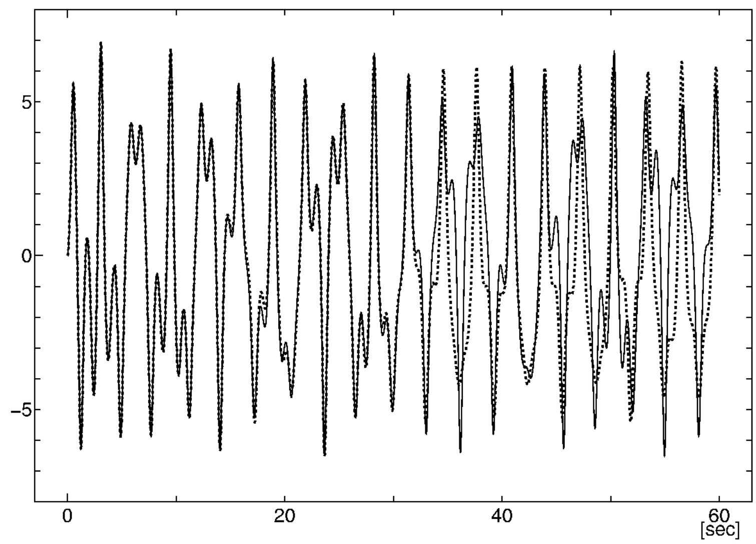

Fig. 11: Comparison of the true and predicted values 2. The case that parameters are estimated from time-series data containing noise. The dotted line corresponds to the actual value and the solid line corresponds to the predicted value. RMSE=1.47052240

References

[1] K.Aihara editor. Chaos -Foundations and applications of chaos theory, Saiensu-sha Co.,Ltd, (1990),(in Japanese, title tentatively translated).

[2] M.Kan and H.Taguchi Capsizing of a Ship in Quatering Seas (Part 2. Chaos and Fractal in Capsizing Phenomenon), *Journal of the Society of Naval Architects of Japan*, **168**, (1990), 211-220.

[3] N.Ohtomo et al,Exponential Characteristics of Power Spectral Densities Caused by Chaotic Phenomena, *Journal of the Physical Society of Japan*, Vol.**64**, Issue 4, pp.1104-1113 (1995).

[4] Y.Ueda et al. Randomly Transitional Phenomena in the System Governed by Duffing's Equation, *Journal of Statistical Physics*, Vol.**20**,Issue 2,(1979), 181-196.

[5] Y.Ueda, Random Phenomena Resulting from Nonlinearity the System Described by Duffing's Equation, *Int.J.Non-Linear Mechanics*, Vol.**20**,No.5-6,(1985), 481-491.

[6] Y.Ueda, Chaos phenomenology, Corona Publishing Co.,Ltd., (2008), (in Japanese, title tentatively translated).

[7] K.Ueno, N.Kimura and K.Amagai, Estimation of Coefficient of the Equation of Nonlinear Roll Motion for Fishing Boats by Improved Energy Method and Genetic Algorithm, *Japan Journal of Industrial and Applied Mathematics* Vol.**20**,No.2,(2003), 155-192.

Received : June 27, 2017
Accepted : August 14, 2017

Academic Backgrounds

Kimihiko UENO

Apr. 1988 - Mar. 1992	Faculty of Fisheries, Hokkaido University, Japan
Apr. 1992 - Mar. 1994	Master's course of Graduate School of Fisheries Science, Hokkaido University, Japan
Apr. 1994 - Jan. 1996	Doctor's course of Graduate School of Fisheries Science, Hokkaido University, Japan
Jan. 1996	Leaving school halfway
Oct. 1997	Ph.D. from Hokkaido University
Apr. 2007 -	Associate Professor, Tokyo University of Marine Science and Technology, Japan

Research Field:
・Complex System and Nonlinear Dynamics for Fishing Boats
・Nonlinear Time Series Analysis for Ship Roll Motion in Irregular Waves
・Parameter Identification for Equation of Motion for Floating Bodies
・Computational Fluid Dynamics for Small Boats

Membership of Professional Institutions:

- The Japan Society for Industrial and Applied Mathematics
- The Japan Society for Mathematical and Physical Fisheries Science
- The Institute of Electronics, Information and Communication Engineers
- The Institute of Systems, Control and Information Engineers
- Mathematics Education Society of Japan
- Japan Society for Natural Disaster Science
- Japan Institute of Navigation

Yoshinobu HIDA

Apr. 2013 - Mar. 2017	Department of Ocean Sciences, Tokyo University of Marine Science and Technology, Japan
Apr. 2017 -	Master's course of Graduate School of Marine Science and Technology, Tokyo University of Marine Science and Technology, Japan

Membership of Professional Institutions:

- The Japan Society for Mathematical and Physical Fisheries Science