Comparative Study of Ocean Wave Height Estimation with Power Spectral Density Function by Three Methods

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Abstract

Since Longuet-Higgins study [7], Rayleigh distribution [13] has been used for ocean wave height analyses in a number of cases. Even if a significant wave height is equal to or less than 2 meter, we cannot ignore its infuence on the work on the deck of small fishing boats. However, there are only few verification examples of such cases. In this study, we estimate wave heights using power spectrum based on Longuet-Higgins method [7] and compare them with actual values for the seakeeping of small fishing boats.

While there are many studies about spectral analysis for ocean waves, there are few studies to evaluate the wave height by the moment of spectral density function. Especially, there are no studies for the difference of evaluation of the wave height caused by the variant of estimation method. There are three major methods for estimations of power spectral density functions. The first one is periodgram method, the second one is correlation function method (e.g. Blackman-Tukey method) and the third one is linear auto regression method (e.g. maximum entropy method). In this paper, we compared estimate results and pointed out problems of estimation results.

1 Introduction

It is important to estimate power spectrum density functions for understanding characteristics of regular waves. In order to obtain their functions, it is necessary to integrate in infinite ranges theoretically. However, it is actually impossible to perform such calculations. Therefore, such power spectrum density functions are obtained only as estimations. Presently, a number of methods are proposed for estimating appropriate spectrum however different characteristics are obtained in each method. Therefore it is necessary to understand characteristics of the estimated power spectrum density functions obtained by each method. While there are many studies about spectral analysis for ocean waves, there are few studies to evaluate the wave height by the moment of spectral density function. Especially, there are no studies for the difference of evaluation of the wave height caused by the variant of estimation methodIn this study, the authors used three major methods that are currently used in a number of studies. The first one is Periodgram Method (P.G.M)[14, 15]. Power spectrum density functions are obtained by direct Fourier transforms of time series data in this method. The second one is Correlation function Method (e.g. Blackman-Tukey method: B.T.M)[3]. Power spectrum density functions are obtained by Fourier transforms of autocorrelation functions in this method. The third one is a method to estimate power spectrum density functions with a linear auto regressive model (e.g. Maximum Entropy Method: MEM)[1, 4]. The authors estimated mean wave periods, mean wave heights, significant wave heights and one tenth maximum wave heights using moments of power spectrum density functions for the sea surface displacement in Tokyo Bay that are estimated by these methods and compared and verified those results and measurement values [9, 10]. From the results, we have obtained the advantages and disadvantages of each method, and discuss the details of them.

2 Analysis Data

Time series data of the sea surface displacements used in this study were measured in Tokyo Bay from 1997 to 2007. One measurement time length was 819.2[sec] and a sampling period was 0.1[sec] (see **Fig.2.1**). We extracted frequency components over 0.1[Hz] and below 1.0[Hz] to remove high frequency noise components with a band-pass filter (see **Fig.A.1** in Appendix 1).

Data used in this study are irregular waves. Therefore, it is necessary to define wave heights in some form. In this study, the authors use Zero-up Crossing Method to define wave heights. In this method, as shown in **Fig.A.2**(see Appendix 1), one wave is defined as an interval between the two points that cross with mean position where the water levels are elevated. The difference between the maximum and minimum values in the period is defined as a wave height in the interval.

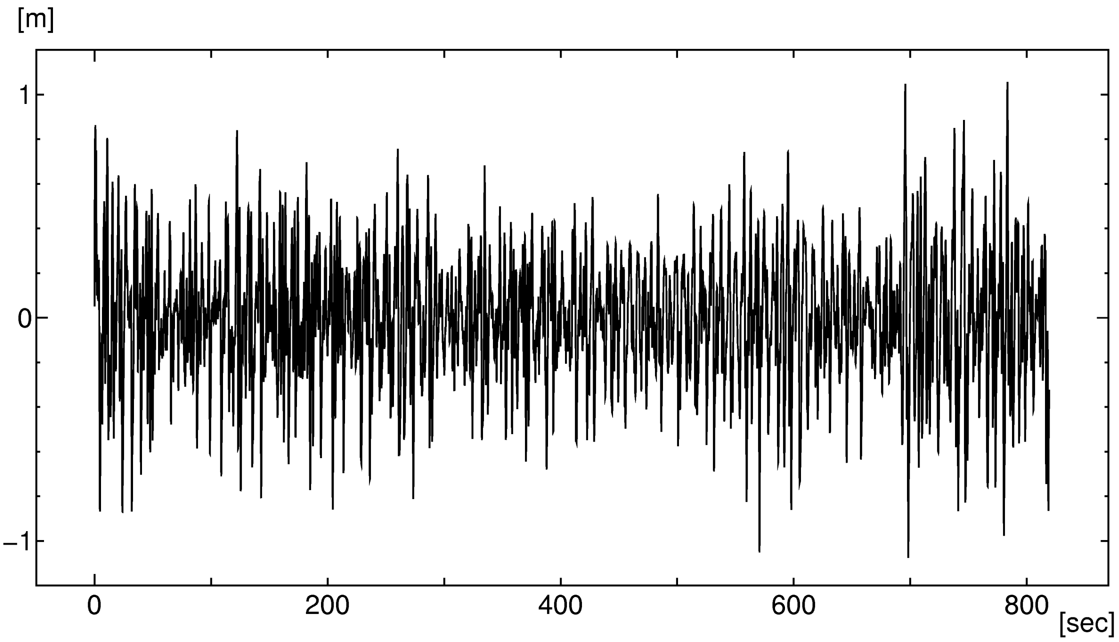

Fig.2.1 Time Series Data of Sea Surface Displacement (Example)

3 Estimation of Power Spectrum Density Function

3.1 PGM:Periodgram Method [14, 15]

In Periodgram Method, Fourier transforms are performed for time series data to obtain to power spectrum density functions. The Fourier transform $Y(f)$ of continuous data $y(t)$ is obtained by the equation below.

$$Y(f) = \int_{-\infty}^{\infty} y(t) e^{-2\pi i f t} dt \tag{1}$$

On the other hand, in this paper, the Fourier transform Y_k of the time series data y_n discretely measured with the sampling period δt is defined by the equation below.

$$Y_k = \sum_{n=1}^{N} y_n e^{-2\pi i f_k n \delta t} \delta t = \delta t \sum_{n=1}^{N} y_n e^{-\frac{2\pi i k n}{N}}, \quad f_k = \frac{k}{N\delta t}, \quad k = 0, 1, \cdots, N \tag{2}$$

Here, N is the total number of data. Moreover, in $-f_N \leqq f_k \leqq f_N, f_N = \frac{1}{2\delta t}$ is Nyquist frequency. The periodgram $S_E(f_k)$ at this time is below

$$S_E(f_k) = |Y_k|^2 \tag{3}$$

The two-sided power spectrum density function $S'_P(f_k)$ is expressed as:

$$S'_P(f_k) = \frac{1}{N\delta t}|Y_k|^2 = \frac{1}{N\delta t} S_E(f_k) \tag{4}$$

Moreover, the one-sided power spectrum density function $S_P(f_k)$ is obtained by doubling the two-sided power spectrum density function $S'_P(f_k)$.

$$S_P(f_k) = 2S'_P(f_k) = \frac{2}{N\delta t}|Y_k|^2 = \frac{2}{N\delta t} S_E(f_k) \tag{5}$$

Positive frequencies are used in ordinary analyses, and therefore discussions in this paper are proceeded with the assumption that the power spectrum density function obtained in PGM is a one-sided power spectrum density function. **Fig.3.1** shows an example of the power spectrum density function obtained by the direct method (PGM).

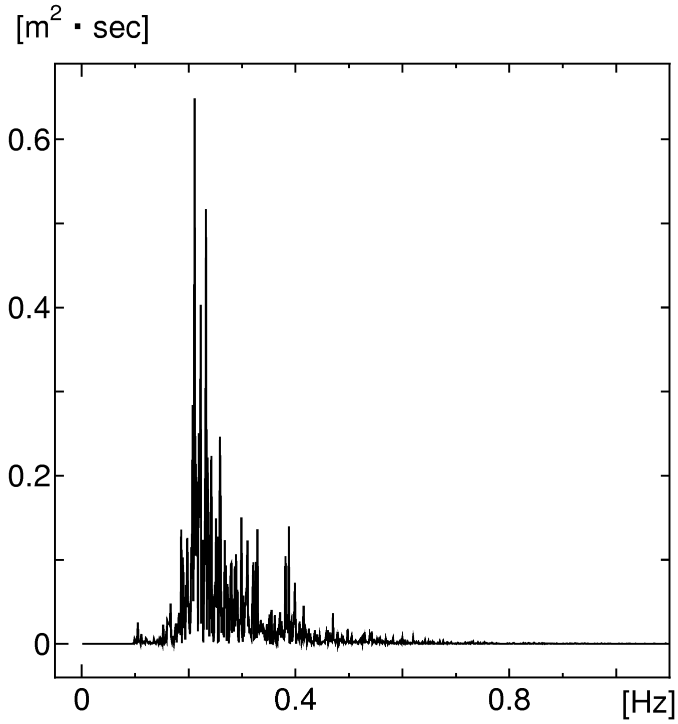

Fig.3.1 Example of Power Spectrum Density Function(PGM) of Sea Surface Displacement

3.2 Correlation Function Method (BTM:Blackman-Tukey Method)[3]

In this method, the autocorrelation function $R(\tau)$ is calculated and then a Fourier transform is performed for it to estimate the power spectrum density function $Sp(\omega)$. This method is based on the Wiener-Khintchine theorem shown below. An angular frequency $\omega(=2\pi f)$ is used for simplification of the calculations

$$\mathfrak{F}[R(\tau)] = \int_{-\infty}^{\infty} R(\tau)e^{-i\omega\tau}d\tau = \int_{-\infty}^{\infty} \left\{ \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} y(t)y(t+\tau)dt \right\} e^{-i\omega\tau}d\tau = S_P(\omega)$$

In this way, a Fourier transformed autocorrelation function is a power spectrum density function and an inverse Fourier transformed power spectrum density function is an autocorrelation function.

In Blackman-Tukey method, which is one of the correlation function methods, an estimation is performed for the data actually obtained based on this relation as below.

The frequency range is assumed $-f_N \leqq f \leqq f_N$ so as to avoid effect of aliasing. Here, f_N is Nyquist frequency and $f_N = \frac{1}{2\Delta t}$. $\triangle\ t$ is a sampling period. Assuming the maximum lag is m, $\triangle\ f = \frac{1}{2m\Delta t}$ therefore $f_k = k \triangle f = \frac{k}{2m\Delta t}$. A power spectrum density function is estimated with the following equation.

$$\begin{aligned} \tilde{S}_P(f_k) &= \tilde{S}_P\left(\frac{kf_N}{m}\right) \\ &= \left\{ \tilde{R}(0) + 2\sum_{i=1}^{m-1} \tilde{R}(i \triangle t)\cos\left(\frac{ki\pi}{m}\right) + \tilde{R}(m \triangle t)\cos(k\pi) \right\} \triangle t \qquad (6) \end{aligned}$$

A Fourier cosine transform is applied for (6) because the autocorrelation function is an even function. **Fig.3.2** shows an example of the power spectrum density function obtained by BTM.

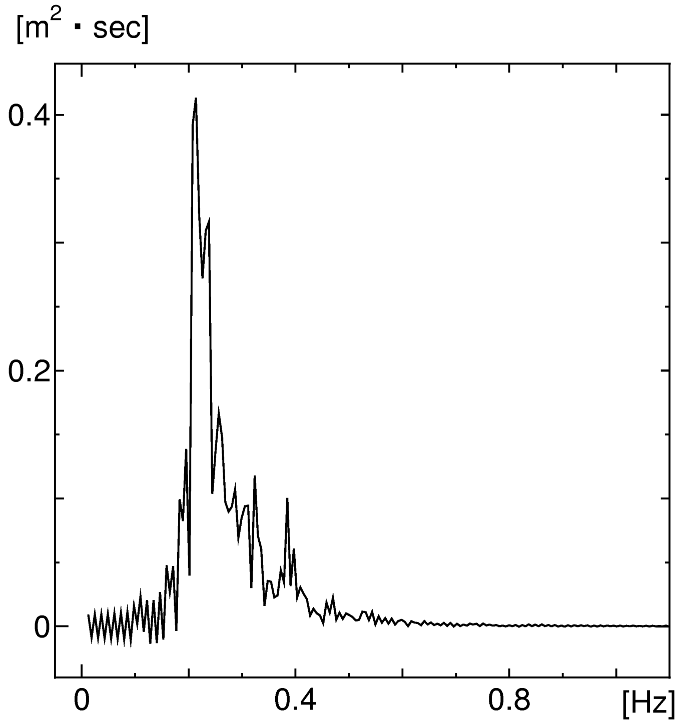

Fig.3.2 Example of Power Spectrum Density Function(BTM) of Sea Surface Displacement

3.3 Linear Auto Regression Method (MEM:Maximum Entropy Method) [1, 4]

Here, we assume that time series data $y(t)(y_i = y(i \bigtriangleup t))$ are presumably expressed in a discrete form as below.

$$y_n = a_1 y_{n-1} + a_2 y_{n-2} + \cdots + a_m y_{n-m} + v_n \tag{7}$$

Here, m is an order of the autoregressive model, a_i is an autoregression coefficient and v_n is white noise independent from the past of y_n for which the mean value is 0 and the variance is σ_m^2.

Assuming that the time series data are based on the autoregressive model expressed by Equation (7), the autocorrelation function $R_k(= R(k \bigtriangleup t))$ is theoretically derived as follows.

$$\begin{aligned} R_k &= R(k \bigtriangleup t) = E[y_n y_{n-k}] \\ &= E\left[\left(\sum_{j=1}^{m} a_j y_{n-j} + v_n\right) y_{n-k}\right] \\ &= \sum_{j=1}^{m} a_j E[y_{n-j} y_{n-k}] + E[v_n y_{n-k}] \end{aligned} \tag{8}$$

Here, from the assumption that v_n is independent from the past of v_n, $E[v_n y_{n-k}] = 0$ for $k > 0$. Moreover, $E[v_n y_n] = \sigma_m^2$. Therefore, following relations are obtained for the autocorrelation function.

$$R_0 = R(0) = \sum_{j=1}^{m} a_j R_j + \sigma_m^2 \tag{9}$$

$$R_k = R(k \bigtriangleup t) = \sum_{j=1}^{m} a_j R_{j-k} \quad (k = 1, 2, \cdots) \tag{10}$$

This is the Yule-Walker's equation [17, 16]. When an autoregressive model of Equation (7) is given, a_j and σ_m^2 are determined. Therefore, an autocorrelation function is obtained by solving Equations (9) and (10) for R_k. A power spectrum density function is given by the following equation then.

$$S_P(f) = \frac{\sigma_m^2}{\left|1 - \sum_{j=1}^{m} a_j e^{-2\pi i f j}\right|^2} \tag{11}$$

For the estimation of the order m, it is proposed to select a method for which AIC(Akaike's Information Criterion)[2]

expressed by the following equation is minimum.

$$\mathrm{AIC}(m) = N \log(2\pi\sigma_m^2) + N + (2m + 1) \tag{12}$$

Here, N is the total number of data. Moreover, it is proposed to maintain the range $m < (2 \sim 3)\sqrt{N}$ so that m with the minimum AIC cannot be great [8].

When estimating an autoregression coefficient by the least squares method, coefficients $(a_1, a_2, \cdots, a_m)$ that minimize the residual sum of squares expressed by Equation (13) are obtained by solving the simultaneous equation (14).

$$S = \sum_{i=m+1}^{N} (y_i - a_1 y_{i-1} - a_2 y_{i-2} - \cdots - a_m y_{i-m})^2 \tag{13}$$

$$\frac{\partial S}{\partial a_j} = 0 \qquad (j = 1, \cdots, m) \tag{14}$$

However, it takes extremely long time to calculate AIC and estimate an optimal order by this method. Therefore, in this study, the authors estimate coefficients by Levinson's recurring formula [11] below.

$$\begin{pmatrix} -1 \\ a_1(m) \\ a_2(m) \\ \vdots \\ a_k(m) \\ \vdots \\ \vdots \\ a_m(m) \end{pmatrix} = \begin{pmatrix} -1 \\ a_1(m-1) \\ a_2(m-1) \\ \vdots \\ a_k(m-1) \\ \vdots \\ a_{m-1}(m-1) \\ 0 \end{pmatrix} - a_m(m) \begin{pmatrix} 0 \\ a_{m-1}(m-1) \\ a_{m-2}(m-1) \\ \vdots \\ a_{m-k}(m-1) \\ \vdots \\ a_1(m-1) \\ -1 \end{pmatrix} \tag{15}$$

Here, $a_k(m)$ indicates a_k in the case that an order of an autoregressive model is m. **Fig.3.3** shows an example of the power spectrum density function obtained from the maximum entropy method (MEM).

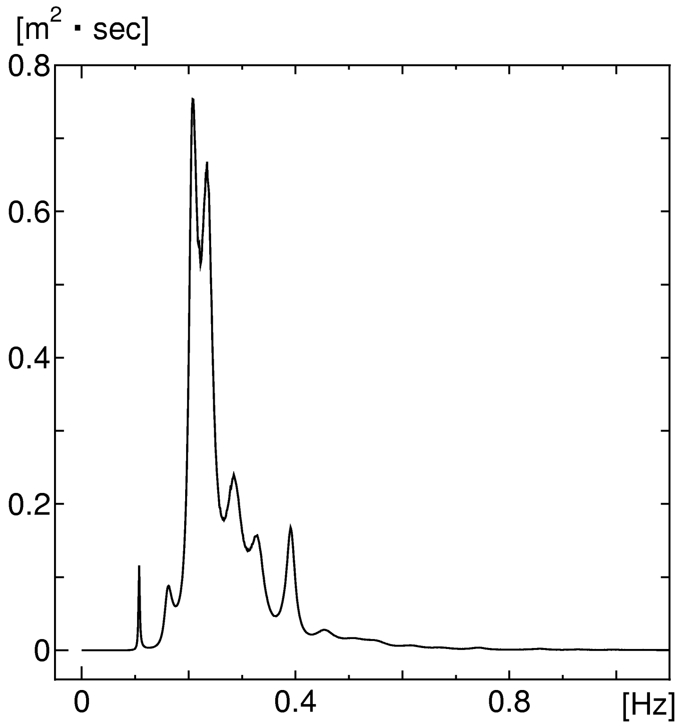

Fig.3.3 Example of Power Spectrum Density Function(MEM) of Sea Surface Displacement

4 Evaluation Method

4.1 Moment of Power Spectrum Density Function

n order moment of a power spectrum density function is defined as the following equation[12].

$$m_n = \int_0^\infty \omega^n S_P(\omega) d\omega \tag{16}$$

For the data that mean value is zero in particular, it is expressed as follows.

$$m_0 = \int_0^\infty S_P(\omega) d\omega = \sigma^2 \tag{17}$$

4.2 Derivation of Mean Wave Period Defined by Zero Up Cross Method with Moment of Power Spectrum Density Function

A square value of the average angular frequency defined by the zero up crossing method with a moment of the spectral density function defined above is expressed as follows.

$$\bar{\omega}^2 = \frac{\displaystyle\int_0^\infty \omega^2 S(\omega) d\omega}{\displaystyle\int_0^\infty S(\omega) d\omega} = \frac{m_2}{m_0} \tag{18}$$

Therefore, mean wave period $\bar{T}$ is expressed as follows.

$$\bar{T} = \frac{2\pi}{\bar{\omega}} = 2\pi\sqrt{\frac{m_0}{m_2}} \tag{19}$$

4.3 $1/n$ Maximum Wave Height in the Case that Rayleigh Distribution is Assumed[13, 7, 12]

We assume a Rayleigh probability density function for wave amplitudes [7, 12] and consider the case that the following equation is assumed.

$$P[x > x_{1/n}] = \int_{x_{1/n}}^\infty p(x) dx = \int_{x_{1/n}}^\infty \frac{x}{m_0} e^{-\frac{x^2}{2m_0}} dx = \frac{1}{n} \tag{20}$$

In this case, $x_{1/n}$ is obtained from the following equation.

$$x_{1/n}^2 = 2m_0 \log_e n \tag{21}$$
$$x_{1/n} = \sqrt{2m_0 \log_e n} \tag{22}$$

The $1/n$ maximum wave amplitude $\bar{x}_{1/n}$ is the x-coordinate of the gravity center of the shaded area. Therefore, it is obtained by dividing the first-order moment by the zero-order moment.(see **Appendix 3**)

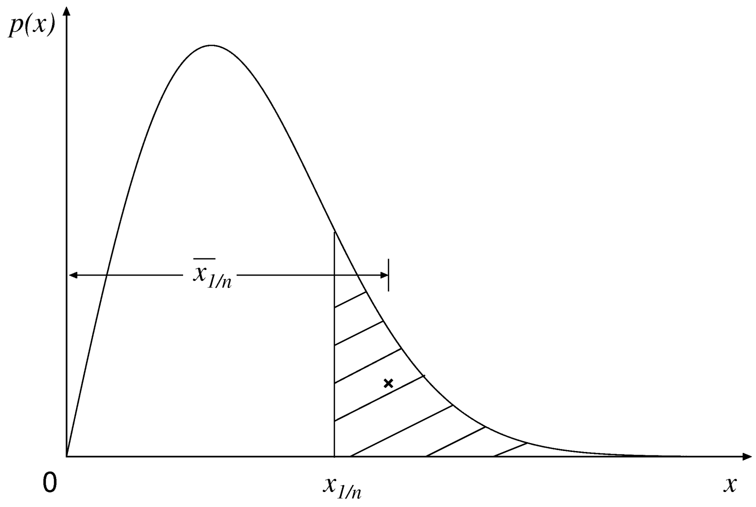

Fig.3.2 Rayleigh Distribution and 1/n Maximum Wave Amplitude

$$\bar{H}_{1/n} \risingdotseq 2\bar{x}_{1/n} = 2\sqrt{2m_0}[\sqrt{\log_e n} + \frac{n\sqrt{\pi}}{2}\{1 - \mathrm{erf}(\sqrt{\log_e n})\}] \tag{23}$$

From the above, when presuming that wave heights of ocean waves are in dependence upon Rayleigh distributions, in the above calculation, the $1/n$ maximum wave height $\bar{H}_{1/n}(\risingdotseq 2\bar{x}_{1/n})$ is estimated from the first-order moment and the zero-order moment of a power spectrum density function. For an approximate calculation of an error function, we used approximate equations of Cody[5] and Hasting[6].

In this study, the authors compare and verify the estimated values obtained from power spectrum density functions with statistics of measured significant wave heights and discuss the validity.

5 Result and Discussion

5.1 Estimated Result of Mean Wave Period

Figures **5.1-5.3** show mean value of wave periods estimated from measurement values of mean wave periods and power spectrum density functions and compare them with scatter diagrams. Results are shown in the order of PGM, BTM and MEM (the following data are shown in the same manner). The solid line in the figure is a regression line of observed values and estimated values and the dotted line is a linear line of $y = x$ (following data are shown in the same manner). In the range shorter than 3[sec], observed values and estimated values agree well with each other for 3 cases. However, estimated values tended to be below observed values longer than 3.0[sec]. In this experimental study, we can not find significant differences due to the estimation method with power spectrum density functions.

5.2 Estimation Result of Mean Value of Wave Height

Figures **5.4-5.6** show mean values of observed wave heights and those estimated from power spectrum density functions and compare them with scatter diagrams. An estimated result by PGM agreed well with the observed values. However, when we evaluated entire errors by Root Mean Square Error (RMSE), we find that estimated results obtained by MEM are best result. In the estimated results obtained by BTM, results are underestimated by 10% for all data comparing with other methods and, moreover, agreement levels are low and RMSE is large.

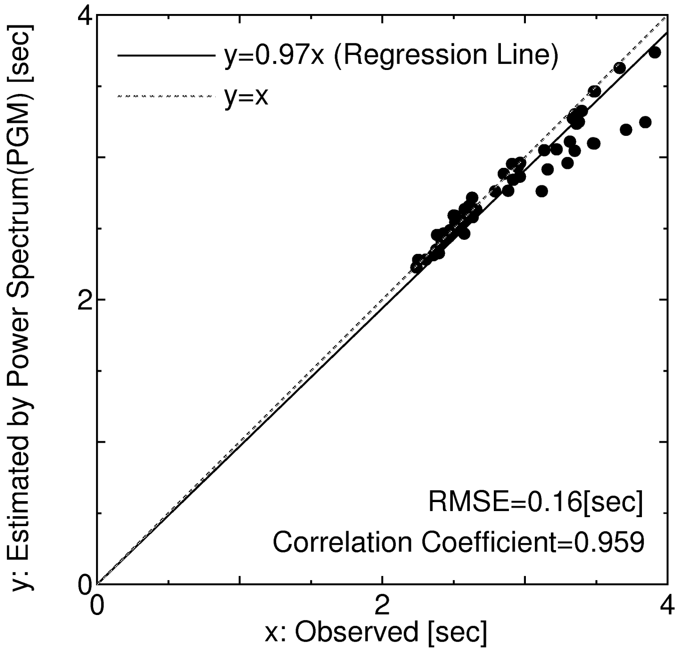

Fig.5.1 Comparison between Observed Value and Estimated Value by Power Spectrum Density Functions (PGM) for Mean Value of Wave Period

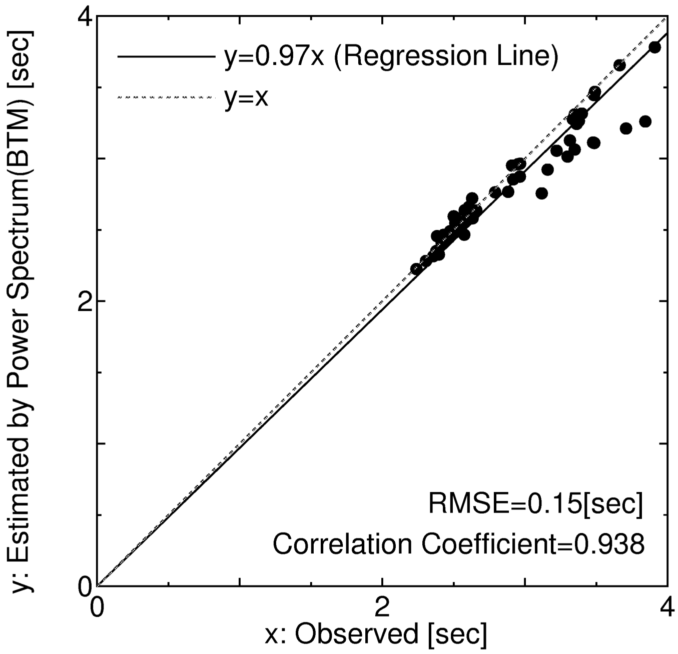

Fig.5.2 Comparison between Observed Value and Estimated Value by Power Spectrum Density Functions (BTM) for Mean Value of Wave Period

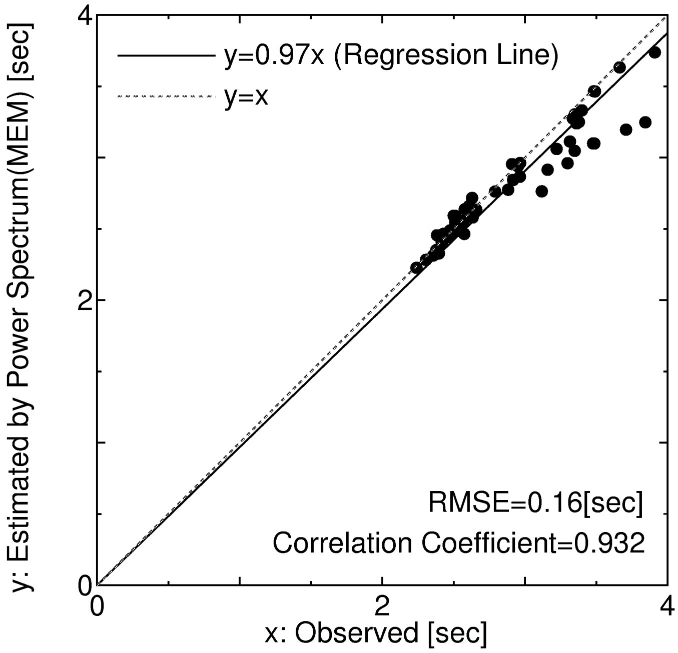

Fig.5.3 Comparison between Observed Value and Estimated Value by Power Spectrum Density Functions (MEM) for Mean Value of Wave Period

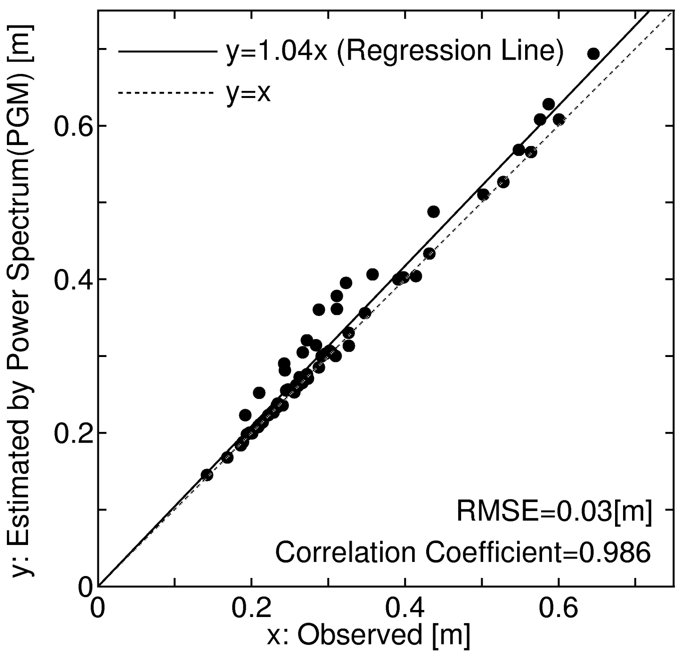

Fig.5.4 Comparison between Observed Value and Estimated Value by Power Spectrum Density Functions (PGM) for Mean Value of Wave Height

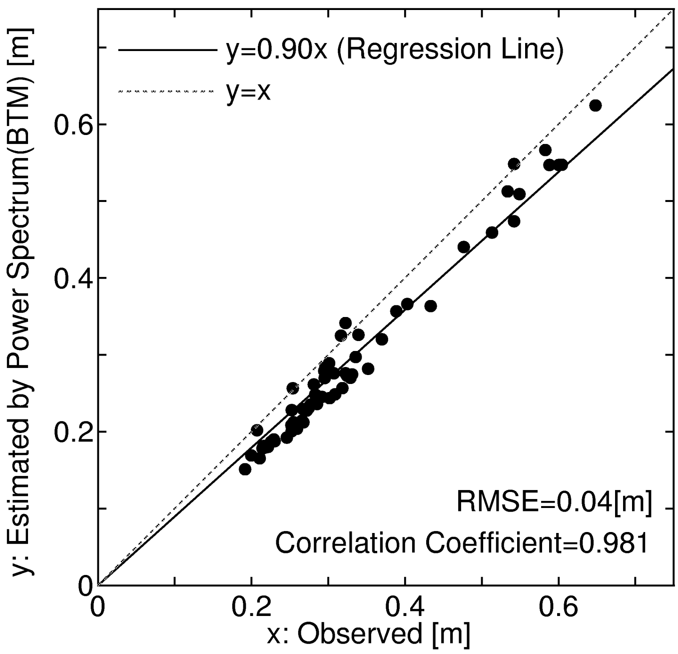

Fig.5.5 Comparison between Observed Value and Estimated Value by Power Spectrum Density Functions (BTM) for Mean Value of Wave Height

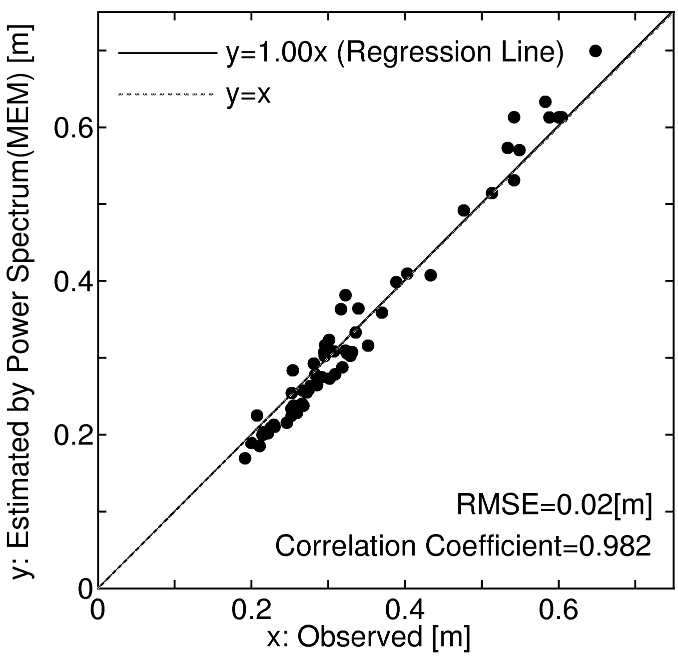

Fig.5.6 Comparison between Observed Value and Estimated Value by Power Spectrum Density Functions (MEM) for Mean Value of Wave Height

5.3 Estimation Result of Significant Wave Height

Figures **5.7-5.9** show observed values of significant wave heights and those estimated from power spectrum density functions and compare them with scatter diagrams.

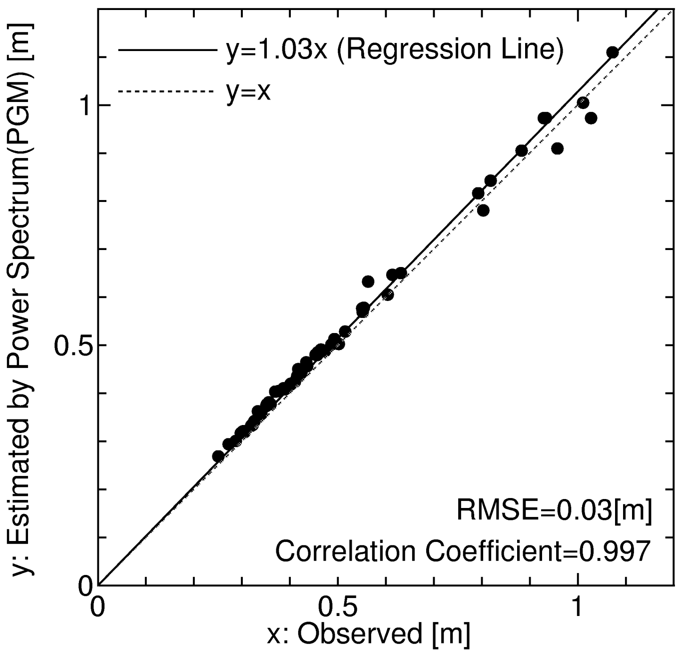

Fig.5.7 Comparison between Observed Value and Estimated Value by Power Spectrum Density Functions (PGM) for Significant Wave Height

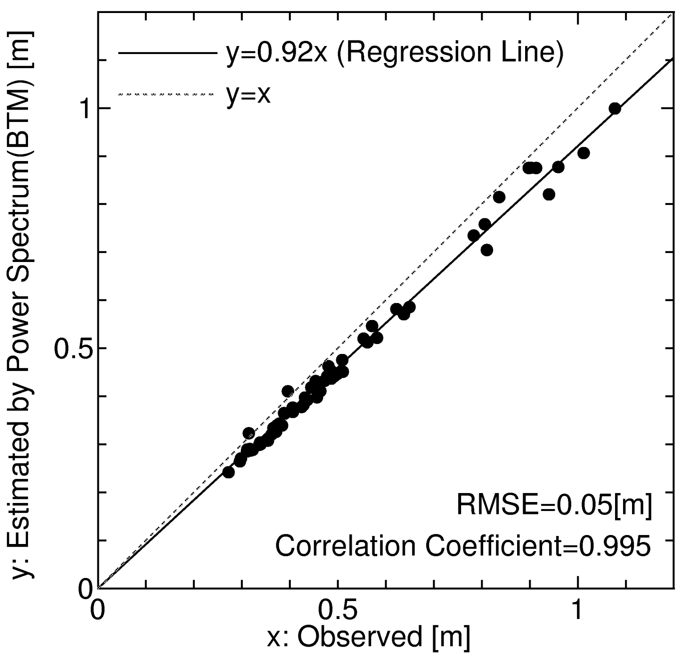

Fig.5.8 Comparison between Observed Value and Estimated Value by Power Spectrum Density Functions (BTM) for Significant Wave Height

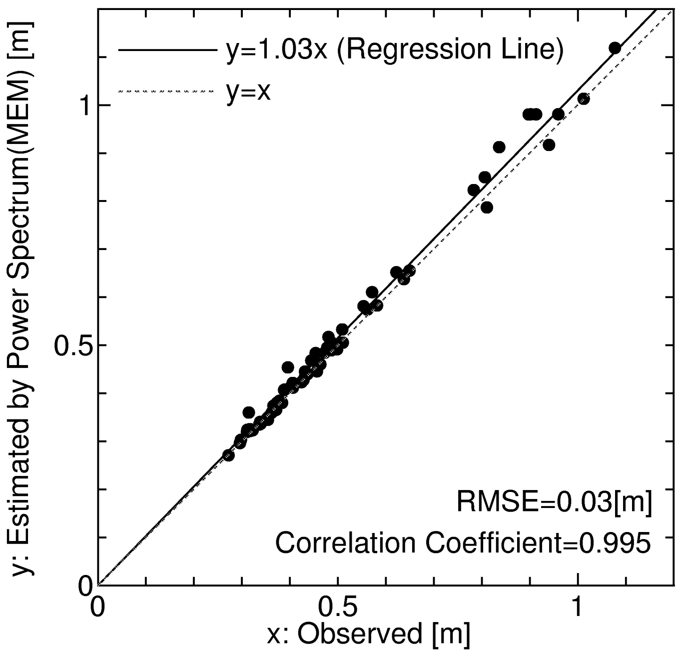

Fig.5.9 Comparison between Observed Value and Estimated Value by Power Spectrum Density Functions (MEM) for Significant Wave Height

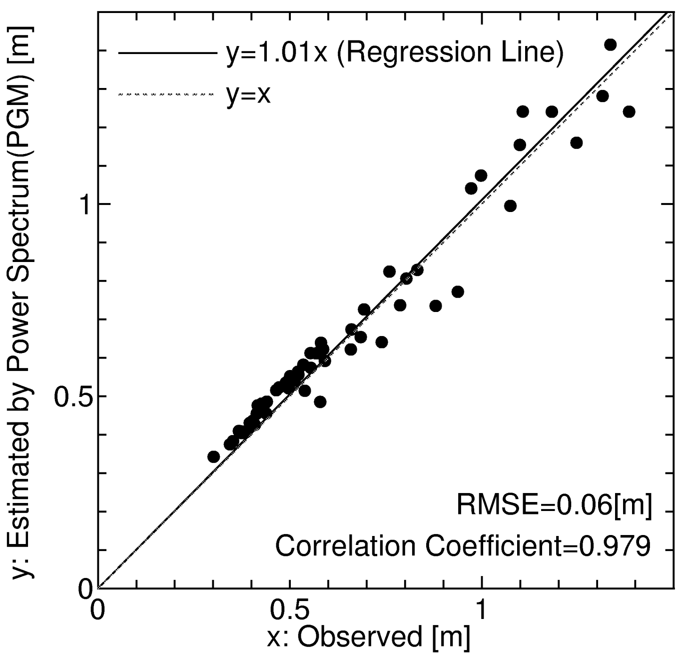

Fig.5.10 Comparison between Observed Value and Estimated Value by Power Spectrum Density Functions (PGM) for 1/10 Maximum Wave Height

5.4 Estimation Result of 1/10 Maximum Wave Height

Finally, Figures **5.10-5.12** show observed values of 1/10 maximum wave heights and those estimated from power spectrum density functions and compare them with scatter diagrams. The agreement is not so good in comparison with both cases of mean wave heights and significant wave heights for all data. RMSE are also greater than the other two cases. This is because the number of waves is fewer than that for both calculations of mean wave heights and significant wave heights. Moreover,in all 3 cases, estimated values in BTM tended to be underestimated for all data.

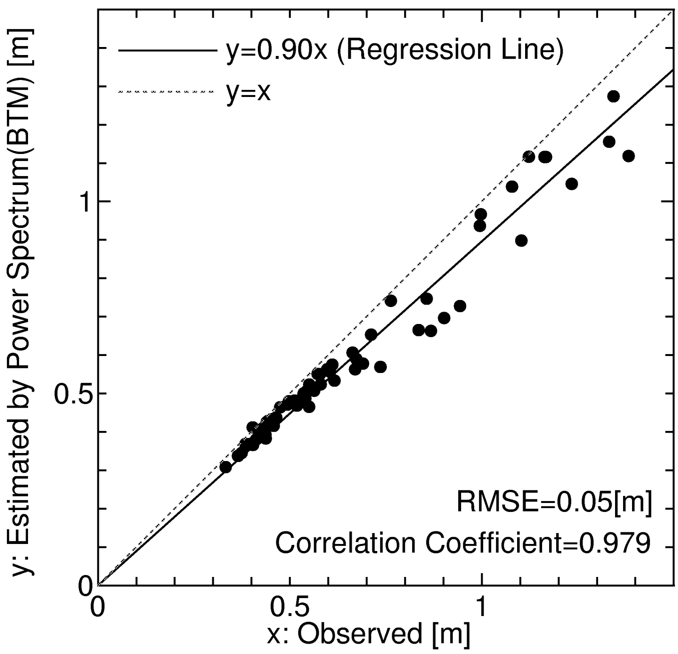

Fig.5.11 Comparison between Observed Value and Estimated Value by Power Spectrum Density Functions (BTM) for 1/10 Maximum Wave Height

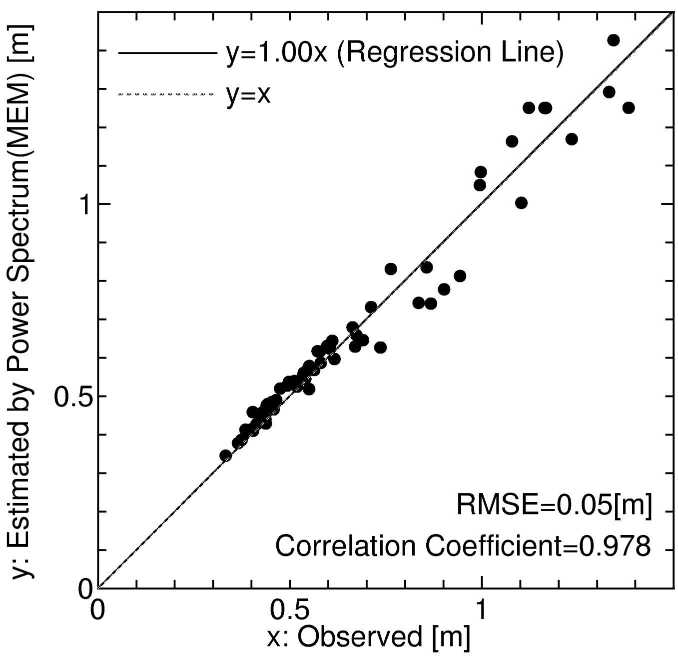

Fig.5.12 Comparison between Observed Value and Estimated Value by Power Spectrum Density Functions (MEM) for 1/10 Maximum Wave Height

6 Conclusion

In the estimation of mean wave periods, the authors do not recognize significant differences due to the difference of calculation methods for power spectrum density functions. Estimated results obtained by PGM are greater by 4% in mean wave heights, 3% in significant wave heights and 1% in 1/10 maximum wave heights than observed values however those differences are acceptable for practical use, we suppose. BTM tends to underestimate by approximately 10% therefore it is necessary to be careful for using this method. Therefore, the authors presume that PGM and MEM are better than BTM for estimations of wave heights. We may have some assumptions about underestimation by BTM. The first one is that the frequency resolution of BTM is inferior to two other methods. Namely, the accuracy of approximate calculation of moments is inferior to two other methods. The second one BTM is based on cosine transformation. Therefore, BTM has a possibility to estimate negative values. Therefore, when we use BTM we need care. Actual phenomena contain a number of uncertainties, however, wave height distributions of ocean waves might be not fairly modeled by the Rayleigh distribution. In this paper, we analyzed the case that the significant wave height was smaller than 1m. However, we are also interest in the case that the significant wave height is larger than 1m. If we have such data, we will analyze them.

References

[1] H.Akaike, Power Spectrum Estimation through Autoregressive Model Fitting, *Annals of the Institute of Statistical Mathematics*, **21** (3) (1969), 407-419.

[2] H.Akaike, A New Look at the Statistical Model Identification, IEEE *Transaction on Automatic Control*, **AC-19** (6) (1974), 716-723.

[3] R.B.Blackman and J.W.Tukey, The Measurement of Power Spectra -From the Point of View of Communication Engineering, American Telephone and Telegraph Company (1958).

[4] J.P.Burg, Maximum Entropy Spectral Analysis, *Geophysics and the Committee on Graduate Studies of Stanford University for the Degree of Doctor of Philosophy* (1975) (http://sepwww.stanford.edu/theses/sep06/).

[5] W.J.Cody, Rational Chebyshev Approximation for the Error Function, *Mathematics of Computation*, **23**(107), (1969), 631-637.

[6] C.Hastings, Jr, Approximations for Digital Computers, Princeton University Press, (1955).

[7] M.S.Longuet-Higgins, On the Statistical Distribution of the Heights of Sea Waves, *Journal of Marine Research*, **XI**(3), (1952), 245-266.

[8] M.Hino, Spectral Analysis, Asakura-syoten (1977) (in Japanese).

[9] H.Kawauchi, K.Ueno, C. Fan, R.Hiraoka and T. Kubokawa, *Mathematical and Physical Fisheries Science*, **6**, (2008), 152-162 (in Japanese).

[10] H.Kawauchi, The Evaluation Comparison of Ocean Wave Height Estimation by using Three Methods, Graduation Thesis of Tokyo University of Marine Science and Technology, (2009) (in Japanese).

[11] H.Levinson, The Wiener RMS (Root Mean Square) Error Criterion in Filter Design and Prediction, *Journal of Mathematics and Physics*, **25** (1947), 261-278.

[12] W.G.Price and R.E.D.Bishop: *Probabilistic Theory of Ship Dynamics*, LONDON CHAPMAN AND HALL(1974)

[13] L.Rayleigh, On the Resultant of a Large Number of Vibration of the Same Pitch and Arbitrary Phase, *Phyilosophical Magazine*, **10**, (1880), 73-78.

[14] A.Schuster, On the Periodicities of Sunspots, *Philosophical Transaction of the Royal Society London. Series A, Containing Papers of a Mathematical and Physical Character* , **206**, (1906), 69-100.

[15] A.Schuster, On the Periodicities of Sun-spots, *Proceeding of the Royal Society London. Series A, Containing Papers of a Mathematical and Physical Character* , **85**(575), (1911), 50-53.

[16] G.Walker, On Periodicity in Series of Related Terms, *Proceedings of the Royal Society of London A*, **131**(1931),518-532.

[17] G.U.Yule, On a Method of Investigating Periodicities in Disturbed Series, *Philosophical Transactions of the Royal Society of London. Series A*, **226**(1927),267-298.

Appendix 1. Filtering and Definition of Wave Height

We extracted frequency components over 0.1[Hz] and below 1.0[Hz] to remove high frequency noise components with a band-pass filter (see **Fig.A.1**).

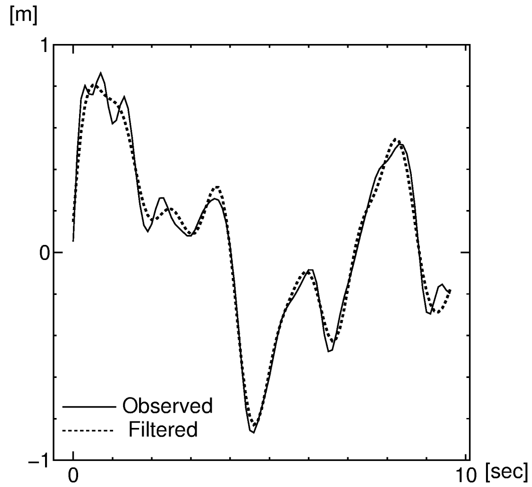

Fig.A.1 Comparison between the Data of Actual Measurement Value and the Data that Passed Filter

In this study, the authors use Zero-up Crossing Method to define wave heights. In this method, as shown in **Fig.A.2**, one wave is defined as an interval between the two points that cross with mean value positions on the parts where the water levels are elevated. The difference between the maximum and minimum values in the period is defined as a wave height in the interval.

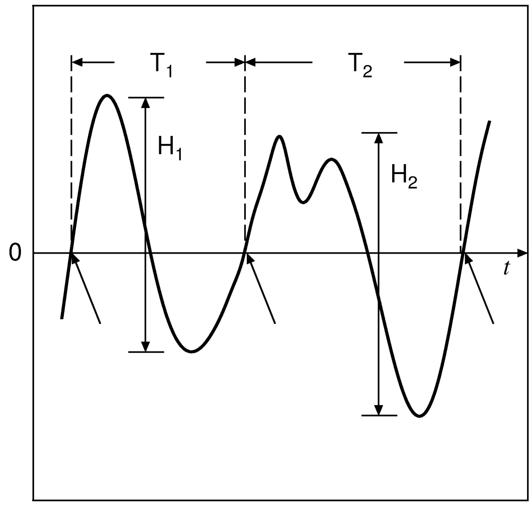

Fig.A.2 Definition of Zero-up Crossing Method

Appendix 2. Wiener-Khintchine Theorem

$$\begin{aligned}
\mathfrak{F}[R(\tau)] &= \int_{-\infty}^{\infty} R(\tau)e^{-i\omega\tau}d\tau = \int_{-\infty}^{\infty}\left\{\lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} y(t)y(t+\tau)dt\right\}e^{-i\omega\tau}d\tau \\
&= \lim_{T\to\infty}\frac{1}{T}\int_{-\infty}^{\infty}\left\{\int_{-\infty}^{\infty} y(t)y(t+\tau)dt\right\}e^{-i\omega\tau}d\tau \\
&= \lim_{T\to\infty}\frac{1}{T}\int_{-\infty}^{\infty} y(t)\left\{\int_{-\infty}^{\infty} y(t+\tau)e^{-i\omega\tau}d\tau\right\}dt \\
&= \lim_{T\to\infty}\frac{1}{T}\int_{-\infty}^{\infty} y(t)\left\{\int_{-\infty}^{\infty} y(t+\tau)e^{-i\omega(t+\tau)}e^{i\omega t}d\tau\right\}dt \\
&= \lim_{T\to\infty}\frac{1}{T}\int_{-\infty}^{\infty} y(t)\left\{\int_{-\infty}^{\infty} y(s)e^{-i\omega s}ds\right\}e^{i\omega t}dt \\
&= \lim_{T\to\infty}\frac{1}{T}\int_{-\infty}^{\infty} y(t)Y(\omega)e^{i\omega t}dt = \lim_{T\to\infty}\frac{1}{T}Y(\omega)\int_{-\infty}^{\infty} y(t)e^{-i(-\omega)t}dt \\
&= \lim_{T\to\infty}\frac{1}{T}Y(\omega)Y(-\omega) = \lim_{T\to\infty}\frac{1}{T}Y(\omega)Y^{*}(\omega) = \lim_{T\to\infty}\frac{1}{T}|Y(\omega)|^2 = S_P(\omega)
\end{aligned}$$

Here, * is a complex conjugate.

Appendix 3. 1/n Maximum Wave Height

$$
\begin{aligned}
\bar{x}_{1/n} &= \frac{\displaystyle\int_{x_{1/n}}^{\infty} xp(x)dx}{\displaystyle\int_{x_{1/n}}^{\infty} p(x)dx} = \frac{\displaystyle\int_{x_{1/n}}^{\infty} xp(x)dx}{\dfrac{1}{n}} \\
&= n\int_{x_{1/n}}^{\infty} xp(x)dx \\
&= n\int_{x_{1/n}}^{\infty} \frac{x^2}{m_0}e^{-\frac{x^2}{2m_0}}dx \\
&= n\int_{x_{1/n}}^{\infty} x(-e^{-\frac{x^2}{2m_0}})'dx \\
&= n\left[-xe^{-\frac{x^2}{2m_0}}\right]_{\sqrt{2m_0\log_e n}}^{\infty} + n\int_{x_{1/n}}^{\infty} e^{-\frac{x^2}{2m_0}}dx \\
&= n\cdot\sqrt{2m_0\log_e n}\cdot\frac{1}{n} + n\int_{\sqrt{\log_e n}}^{\infty} e^{-t^2}(\sqrt{2m_0}dt) \\
&= \sqrt{2m_0\log_e n} + n\sqrt{2m_0}\cdot\frac{\sqrt{\pi}}{2}(\frac{2}{\sqrt{\pi}}\int_0^{\infty} e^{-t^2}dt - \frac{2}{\sqrt{\pi}}\int_0^{\sqrt{\log_e n}} e^{-t^2}dt) \\
&= \sqrt{2m_0\log_e n} + n\sqrt{2m_0}\cdot\frac{\sqrt{\pi}}{2}\{1-\mathrm{erf}(\sqrt{\log_e n})\} \\
&= \sqrt{2m_0}[\sqrt{\log_e n} + \frac{n\sqrt{\pi}}{2}\{1-\mathrm{erf}(\sqrt{\log_e n})\}] \fallingdotseq \frac{1}{2}\bar{H}_{1/n}
\end{aligned}
$$

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