Calculation of the hydrodynamic pressures acting on catamaran's cross sections with wave interaction

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Abstract

In the design of a fishing catamaran, we must make sure of the strength of her cross deck structure. Some existing researches showed the most severe condition for that strength is in that the ship stops in beam seas. In such a condition, the strip method is still effective, but it must be used the hydrodynamic forces and moments including the influence of the wave interaction between the hulls. And a local structural analysis by F.E.M. requires not only forces and moments but also pressure on the hull.

In this paper, the author computed hydrodynamic forces, moments and pressures acting on a catamaran's cross section by a method proposed by the author [7]. The paper especially focuses into the influence of a local wave component in the interaction waves.

1 Introduction

By historical rise in crude oil prices last year, further fuel saving is required for Japanese fishing vessels. Hull form improvement is a conventional way to achieve the purpose. But on the other hand, it is necessary for fishing vessels to develop the technologies utilizing natural energy. Wave devouring propulsion system is one of them.

A wave devouring propulsion system that converts ocean wave energy into ship driving power is one of the devices utilizing natural energy for ships. In 2008, the famous ocean adventurer Kenichi Horie has sailed from Hawaii to Japan with his catamaran type wave power boat "Mermaid II" by only the wave devouring propulsion system [1]. Comparing a mono hull ship and a catamaran, the latter is superior to the former as the basic hull form for the wave power boat. The reason is probably that the pitching of a catamaran in waves is generally much larger than that of a mono hull ship. Furthermore, using the dual wing type wave devouring propulsion system [2], the converting efficiency wave energy into the thrust reaches maximum in beam seas.

However, according to Hadler et al. [3], the most severe condition for the lateral

strength of the cross deck structure of a catamaran is also stopping in beam seas. Then, the accurate prediction of hydrodynamic forces acting on a catamaran is necessary for both ship motion prediction and confirmation of her lateral strength.

The reference [4] has shown that the strip method is still effective taking account of the influence of the wave interaction between hulls. And for the analysis of the local strength of the cross deck structure by the F.E.M., pressure on the hull is required as the external force imposing to a ship structural model. Pressure is commonly calculated as the solution of the boundary value problem for twin hull section [5]. Ohkusu's method can calculate the forces and moments of twin hull section from the solution of a single hull section but it cannot calculate the pressures [6].

The author has already proposed new two pressure prediction methods for a twin hull section oscillating in beam seas [7]. The methods can compute pressures including the influence of wave interaction between hulls from the radiation pressures of a single hull section like as Ohkusu's method. One method is the 2D version of the unified slender body theory for a catamaran [8], [9] and it was called "Method1" in the reference [7]. Another method is the extension of the Ohkusu's method utilizing the Bessho's relation for the time reversed potential theory [10], [11] and it was called "Method2" [7]. By using these methods, predicted ship motions and sea loads have shown good agreement with experiments [12].

But in the reference [7] and [12], the author omitted the interaction terms of local wave component in computations. So this paper describes the outline of the "Method1" and focuses into the influence of the local wave terms in computational results.

2 Formulation

2.1 Radiation problems for a single hull section

In this section, we describe variable definitions and some theorems of 2D hydrodynamics for a single hull section related to formulate the "Method1" of pressures of a twin hull section. In subsequent descriptions, we will use the following notations: the single overbar is for a complex conjugate, the symbol Re is for the real part and Im is for the imaginary part.

We introduce a coordinate system $o-xyz$ with the origin placed at the center line of the hull section and on the undisturbed free surface as shown in Fig.1. C is the girth of the section, g is the acceleration of the gravity and i is the unit of an imaginary number, ω is the wave angular frequency, $K=\omega^2/g$ is the wave number.

2D velocity potentials ϕ_j for the j-mode fluid motion (j =1: sway, 2: heave, 3: roll,

4: diffraction) for a single hull section can be expressed as

$$\phi_j(y,z)\,\mathrm{e}^{i\omega t} = \int_C \left(\frac{\partial \phi_j}{\partial n} - \phi_j \frac{\partial}{\partial n} \right) S(y,z;y',z')\,dC\,\mathrm{e}^{i\omega t} \tag{1}$$

$$S(y,z;y',z') = \frac{1}{2\pi}\ln\left(\frac{r}{r'}\right) - \lim_{\mu\to+0}\frac{1}{\pi}\int_0^\infty \frac{\mathrm{e}^{-l(z+z')}\cos\{l(y-y')\}}{l-K+i\mu}\,dl \tag{2}$$

where, $r=\sqrt{(y-y')^2+(z-z')^2}$, $r'=\sqrt{(y-y')^2+(z+z')^2}$. S defined by (2) is the linear source function and μ is the Rayleigh's fictitious friction factor. $n=(n_y,n_z)$ is the outward unit normal vector on the girth C. Here, for simplicity, we introduce another vector n_j corresponding to j-mode motion defined as

$$(n_1,n_2,n_3) = (n_y, n_z, n_z y - n_y z) \tag{3}$$

Each ϕ_j should be satisfied a following hull boundary condition.

$$\frac{\partial \phi_j}{\partial n} = i\,\omega\,\xi_j\,n_j\,,\ (\,j=1\sim3),\quad \frac{\partial \phi_4^{\pm}}{\partial n} = -\frac{\partial \phi_0^{\pm}}{\partial n} \tag{4}$$

Where, ξ_j is the j-mode motion amplitude and $\phi_0^{\pm}$ is an incident wave velocity potential incoming from ± direction along y axis.

We introduce the the unit amplitude velocity potentials φ_j coresponding to ϕ_j defined as

$$\phi_j(y,z) = i\,\omega\,\xi_j\,\varphi_j(y,z)\,,\ (\,j=1\sim3),\quad \phi_4^{\pm}(y,z) = \frac{g\zeta_w}{i\omega}\varphi_4^{\pm}(y,z) \tag{5}$$

In the same way,

$$\phi_0^{\pm}(y,z) = \frac{g\zeta_w}{i\omega}\varphi_0^{\pm}(y,z)\,,\quad \varphi_0^{\pm}(y,z) = \mathrm{e}^{-K(z\pm iy)} \tag{6}$$

Where, $\varphi_0^{\pm}$ are the coresponding unit amplitude incident wave velocity potentials and ζ_w is the wave amplitude.

A scattering potentials $\varphi_s^{\pm}$ are defined as

$$\varphi_s^{\pm} = \varphi_0^{\pm} + \varphi_4^{\pm} \tag{7}$$

The hull boundary conditions for φ_j and $\varphi_4^{\pm}$ corresponding to (4) are witten as

$$\frac{\partial \varphi_j}{\partial n} = n_j\,,\ (\,j=1\sim3),\quad \frac{\partial \varphi_4^{\pm}}{\partial n} = -\frac{\partial \varphi_0^{\pm}}{\partial n} \tag{8}$$

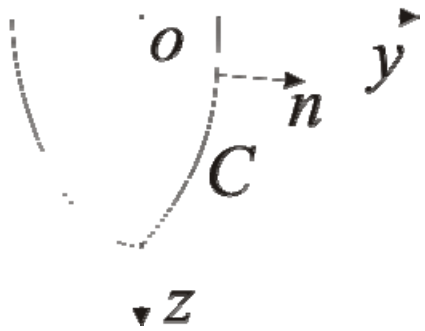

Fig.1 Coordinate system and variable definitions for a mono-hull section

By the way, according to Bessho's theorem for time-reversed velocity potentials [8], following Bessho's relation has held between the radiation potential φ_j and the scattering potential $\varphi_s^{\pm}$.

$$\overline{\varphi}_j = \varphi_j - i(\overline{h}_j^{+}\varphi_s^{+} + \overline{h}_j^{-}\varphi_s^{-}),\ (j=1\sim3) \tag{9}$$

$\overline{\varphi}_j$ is a time-reversed velocity potential and also the complex conjugate of φ_j, and $\overline{h}_j^{\pm}$ is the complex conjugate of $h_j^{\pm}$ that is the Kochin functions defined as follows.

$$h_j^{\pm} = \sigma_j \pm i\mu_j = \int_C \left(\frac{\partial \varphi_j}{\partial n} - \varphi_j \frac{\partial}{\partial n} \right) \mathrm{e}^{-K(\zeta z' \mp iy')}\, ds \tag{10}$$

The double sign $\pm$ denotes the direction of outgoing wave along y axis. The source strength σ_j and the doublet moment μ_j for the j-mode fluid motion can be calculated by the Kochin functions as

$$\sigma_j = \frac{1}{2}(h_j^{+} + h_j^{-}),\quad \mu_j = \frac{1}{2i}(h_j^{+} - h_j^{-}) \tag{11}$$

We consider a simultaneous equation consisted of (9) for the j-mode and another k-mode radiation potentials and solve it for $\varphi_s^{\pm}$, we obtain a following formula [8], [9].

$$\varphi_s^{\pm} = \frac{\overline{h}_k^{\mp}(\varphi_j - \overline{\varphi}_j) - \overline{h}_j^{\mp}(\varphi_k - \overline{\varphi}_k)}{i(\overline{h}_j^{\mp}\overline{h}_k^{\pm} - \overline{h}_j^{\pm}\overline{h}_k^{\mp})} \tag{12}$$

This formula shows the scattering potentials can be repesented by the radiation potentials and their Kochin functions. Therefore, by using this formula, we can calculate the scattering potential $\varphi_s^{\pm}$ from the radiation potentials φ_j, φ_k by only algebraic computation. This is one of the essential results of Bessho's time-reversed velocity potential theory.

Similar relations as (9) have held between the Kochin functions.

$$\overline{h}_j^{-} = h_j^{+} - i(\overline{h}_j^{+}h_4^{+} + \overline{h}_j^{-}h_4^{-}),\ (j=1\sim3) \tag{13}$$

And we obtain a following formula for the Kochin function $h_4^{\pm}$ of the diffraction potential as

$$h_4^{\pm} = \pm i \frac{\overline{h}_k^{\mp}(\overline{h}_j^{-} - h_j^{+}) - \overline{h}_j^{\mp}(\overline{h}_k^{-} - h_k^{+})}{\overline{h}_j^{+}\overline{h}_k^{-} - \overline{h}_j^{-}\overline{h}_k^{+}} \tag{14}$$

Pressure on the girth is calculated by a following equation.

$$p_j(y,z)\,\mathrm{e}^{i\omega t} = \rho i\,\omega\xi_j \frac{\partial \phi_j(y,z)\,\mathrm{e}^{i\omega t}}{\partial t} = -\rho\omega^2 \xi_j \phi_j(y,z)\,\mathrm{e}^{i\omega t} \tag{15}$$

Radiation forces and moments f_{ij} can be calculated as

$$f_{ij} = \int_{C_R} n_i\, p_j(y,z)\, ds = i\omega\xi_j(i\omega a_{ij} + b_{ij}) \tag{16}$$

Where, a_{ij} is an added mass and b_{ij} is damping coefficient and is obtained as

$$a_{ij} = -\mathrm{Re.}\left[\frac{f_{ij}}{\omega^2 \xi_j}\right],\ b_{ij} = \mathrm{Im.}\left[\frac{f_{ij}}{\omega \xi_j}\right] \tag{17}$$

Wave exciting forces and moments $e_j^{\pm}$ can be calculated by the Haskind relation as following.

$$e_j^{\pm} = -\rho g \zeta_w h_j^{\pm} \tag{18}$$

2.2 Radiation problems for a twin hull section

In this chapter, we describe a computing method of the radiation pressures of a twin hull section from that of a single hull section. The method was called "method-1" in reference [7] that has been derived as a high frequency version of the unified slender body theory for a catamaran [10], [11].

We introduce coordinate systems as shown in Fig.2 and focus to the flow field near the left hull. We assume that the each hull section may be asymmetric to z axis, but the twin hull section is symmetric to Z axis. The flow field near the right hull can be computed from that of the left hull by using symmetry or anti-symmetry of the entire flow field.

The hull boundary conditions for φ_j and $\varphi_4^{\pm}$ corresponding to (4) are witten as and its unit amplitude velocity potential ψ_j. ψ_j should be satisfied following hull boundary condition on C_L.

$$\Psi_j(Y,Z)\mathrm{e}^{i\omega t} = i\omega\xi_j\psi_j(Y,Z)\mathrm{e}^{i\omega t},\ \frac{\partial\psi_j}{\partial N} = N_j,\ (j=1\sim3) \tag{19}$$

Where, $N = (N_1, N_2, N_3)$ is the outward nomal vector on C_L defined to the coordinate system $O-Y,Z$, and N_j is written by the unit nomal vector n_j to the coordinate system $o-y,z$ defined by (3) as following.

$$N_1 = n_1,\ N_2 = n_2,\ N_3 = n_3 \mp \frac{P}{2} n_2 \ \text{ on } \begin{cases} C_L \\ C_R \end{cases} \tag{20}$$

where; P is the distance between hulls.

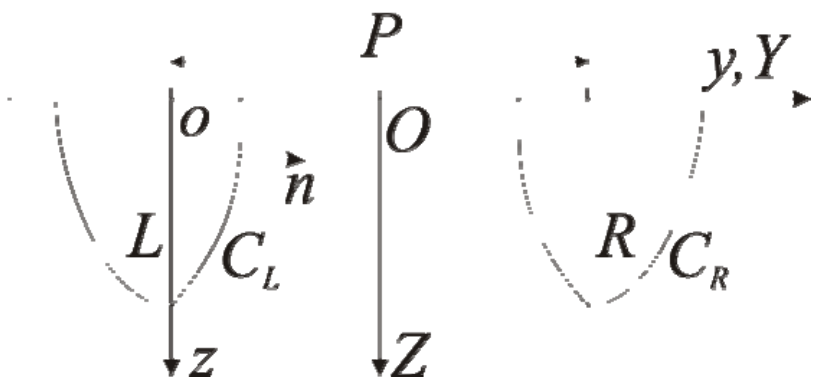

Fig.2 Coordinate systems and variable definitions for a twin hull section

We focus into the flow field near the left hull and write the velocity potentials of the twin hull section as follows.

$$\psi_j(y,z) = \varphi_j(y,z) + C_j\{\varphi_2(y,z) - \overline{\varphi}_2(y,z)\} + D_j\{\varphi_1(y,z) - \overline{\varphi}_1(y,z)\} \tag{21}$$

ψ_j consists of the velocity potential as a single hull section and two homogeneous potentials multiplied two unknown interaction coefficients to determine symmetric and anti-symmetric flow field independently. The unknown interaction coefficients C_j and D_j will be determined by the matching with a far field velocity potential.

Here, we decompose the velocity potential φ_j into the symmetric flow component P_j and the anti-symmetric component Q_j as following.

$$\varphi_j(y,z) = \sigma_j P_j(y,z) + \mu_j Q_j(y,z) \tag{22}$$

$$P_j(y,z) = G(y,z) + \text{wave free potential} \tag{23}$$

$$Q_j(y,z) = H(y,z) + \text{wave free potential} \tag{24}$$

G is the wave source potential and H is the wave doublet potential with horizontal axis and these are represented as follows.

$$G(y,z) = -\lim_{\mu\to+0}\frac{1}{\pi}\int_0^\infty \frac{\mathrm{e}^{-nz}\cos(ny)}{n-K+i\mu}dn = -\frac{1}{\pi}\mathrm{Re.}\left[\mathrm{e}^{-K(z-iy)}E_1\{-K(z-iy)\}\right] + i\,\mathrm{e}^{-K(z+i|y|)}$$

$$(25)\ H(y,z) = -\frac{1}{K}\frac{\partial}{\partial y}G(y,z) = -\frac{1}{\pi}\mathrm{Im.}\left[\mathrm{e}^{-K(z-iy)}E_1\{-K(z-iy)\} + \frac{1}{K(z-iy)}\right] - \mathrm{e}^{-K(z+i|y|)} \tag{26}$$

Where, E_1 denotes the exponential integral defined as follows.

$$E_1(z) = \int_z^\infty \frac{e^{-t}}{t}dt \tag{27}$$

Substituting (22) ~ (24) into (21),

$$\psi_j(y,z) = \sigma_j P_j(y,z) + C_j\{\sigma_2 P_2(y,z) - \overline{\sigma}_2\overline{P}_2(y,z)\} + C_j\{\mu_2 Q_2(y,z) - \overline{\mu}_2\overline{Q}_2(y,z)\}$$
$$+\mu_j Q_j(y,z) + D_j\{\sigma_1 P_1(y,z) - \overline{\sigma}_1\overline{P}_1(y,z)\} + D_j\{\mu_1 Q_1(y,z) - \overline{\mu}_1\overline{Q}_1(y,z)\} \tag{28}$$

we decompose (28) into symmetric and anti-symmetric flow components.

$$\psi_j(y,z) = \sigma_j P_j + C_j(\sigma_2 - \overline{\sigma}_2)P_2 + D_j(\sigma_1 - \overline{\sigma}_1)P_1 + C_j\overline{\sigma}_2(P_2 - \overline{P}_2) + D_j\overline{\sigma}_1(P_1 - \overline{P}_1)$$
$$+\mu_j Q_j + C_j(\mu_2 - \overline{\mu}_2)Q_2 + D_j(\mu_1 - \overline{\mu}_1)Q_1 + C_j\overline{\mu}_2(Q_2 - \overline{Q}_2) + D_j\overline{\mu}_1(Q_1 - \overline{Q}_1) \tag{29}$$

Th high frequency expansions of P_j and Q_j under the assumption of $Kr_0 = O(1)$ as $r_0 = \sqrt{y^2 + z^2}$, (23) and (24) can be expanded as

$$P_j(y,z) \approx G(y,z) \approx \frac{1}{\pi}\frac{Kz}{(Kr_0)^2} + i\,\mathrm{e}^{-K(z+iy)} \tag{30}$$

$$Q_j(y,z) \approx H(y,z) \approx -\mathrm{e}^{-K(z+iy)} \tag{31}$$

And the homogeneous components are written as

$$P_j(y,z) - \overline{P}_j(y,z) = 2i\,\mathrm{e}^{-Kz}\cos Ky \tag{32}$$

$$Q_j(y,z) - \overline{Q}_j(y,z) = 2i\,\mathrm{e}^{-Kz}\sin Ky \tag{33}$$

Substituting (30) ~ (33) into (29), we obtain the followimg far field expansion of (21).

$$\begin{aligned}\psi_j(y,z) \approx\ & \{\sigma_j + C_j(\sigma_2 - \overline{\sigma}_2) + D_j(\sigma_1 - \overline{\sigma}_1)\}G(y,z) + 2i\,(C_j\overline{\sigma}_2 + D_j\overline{\sigma}_1)\,\mathrm{e}^{-Kz}\cos Ky \\ & + \{\mu_j + C_j(\mu_2 - \overline{\mu}_2) + D_j(\mu_1 - \overline{\mu}_1)\}H(y,z) + 2i(C_j\overline{\mu}_2 + D_j\overline{\mu}_1)\,\mathrm{e}^{-Kz}\sin Ky \end{aligned} \tag{34}$$

The expansion under the assumption of $Kr_0 = O(\varepsilon)$ can be obtained as a Taylor exansion of (34) for Ky and Kz as following.

$$\begin{aligned}\psi_j(y,z) \approx\ & \{\sigma_j + C_j(\sigma_2 - \overline{\sigma}_2) + D_j(\sigma_1 - \overline{\sigma}_1)\}G(y,z) + 2i(C_j\overline{\sigma}_2 + D_j\overline{\sigma}_1)(1 - Kz) \\ & + \{\mu_j + C_j(\mu_2 - \overline{\mu}_2) + D_j(\mu_1 - \overline{\mu}_1)\}H(y,z) + 2i(C_j\overline{\mu}_2 + D_j\overline{\mu}_1)Ky \end{aligned} \tag{35}$$

(34) and (35) are understood as the far field expansion of the near field solution.

Next, we consider a far field solution. The far filed disturbance from the twin hull section is expressed by two couples of a point source and doublet located at the origins of the left and right hull section. Taking account of symmetry or anti-symmetry of the flow field depending on the oscillation mode, the far field solution is expressed as

$$\psi_j(y,z) \approx \Sigma_j G(y,z) + M_j H(y,z) + (-1)^j\{\Sigma_j G(y-P,z) - M_j H(y-P,z)\} \tag{36}$$

Where, Σ_j is the source strength and M_j is the doublet moment of the left hull section. Σ_j and M_j are unknown and will determine by the matching with the far field expansion of the near field solution.

Near the left hull section, we can assume $r_0 / P << 1$, then the source and doublet functions can be expanded as

$$G(y-P,z) \approx (1-Kz)g_s(P) + Kyg_a(P) \tag{37}$$

$$H(y-P,z) \approx (1-Kz)h_s(P) + Kyh_a(P) \tag{38}$$

Where, the functions g_s, g_a, h_s and h_a are defined as follows.

$$g_s(P) = -\frac{1}{\pi}\mathrm{Re.}\left[\mathrm{e}^{-iKP}E_1(-iKP)\right] + i\,\mathrm{e}^{-iKP} \tag{39}$$

$$g_a(P) = -h_s(P) = \frac{1}{\pi}\mathrm{Im.}\left[\frac{1}{iKP} + \mathrm{e}^{-iKP}E_1(-iKP)\right] - \mathrm{e}^{-iKP} \tag{40}$$

$$h_a(P) = \frac{1}{\pi}\mathrm{Re.}\left[\frac{1}{(KP)^2} - \mathrm{e}^{-iKP}E_1(-iKP)\right] + i\,\mathrm{e}^{-iKP} = \frac{1}{\pi}\frac{1}{(KP)^2} + g_s(P) \tag{41}$$

The terms in the bracket of (39)～(41) express the local waves and the other terms express the outgoing waves. These equations show the influence of the local wave becomes larger for lower frequency or shorter distance between hull sections.

From (36) ~ (38), we obtain the near field expansion of the far field solution as

$$\begin{aligned}\psi_j(y,z) =\ & \Sigma_j G(y,z) + (-1)^j\Sigma_j\{(1-Kz)g_s + Kyg_a\} \\ & + M_j H(y,z) - (-1)^j M_j\{(1-Kz)h_s + Kyh_a\}\ ,\ (j = 1～3) \end{aligned} \tag{42}$$

Comparing (35) and (36), and taking account of the terms in (34) approximated as

$$\mathrm{e}^{-Kz}\cos Ky \approx 1-Kz\,,\ \ \mathrm{e}^{-Kz}\sin Ky \approx Ky \tag{43}$$

We obtain matching conditions as a following simultaneous equation to determine the unknown constant C_j, D_j, Σ_j and M_j.

$$\sigma_j + C_j(\sigma_2-\overline{\sigma}_2)+D_j(\sigma_1-\overline{\sigma}_1)=\Sigma_j\,,\ \ \mu_j + C_j(\mu_2-\overline{\mu}_2)+D_j(\mu_1-\overline{\mu}_1)=M_j \tag{44}$$

$$2i(C_j\overline{\sigma}_2+D_j\overline{\sigma}_1)=(-1)^j(\Sigma_j g_s - M_j h_s)\,,\ \ 2i(C_j\overline{\mu}_2+D_j\overline{\mu}_1)=(-1)^j(\Sigma_j g_a - M_j h_a) \tag{45}$$

Solving (44) and (45) by algebraic or numerical method, substituting C_j, D_j, Σ_j and M_j into (21), finally we get the radiation potentials of twin hull section including the influence of interaction waves between hulls. But φ_j only satisfy the hull boundary condition (8) of the single hull section. For the cases of the sway velocity potential ψ_1 and heave's ψ_2, there are no difference the normal vectors between (3) and (20). However, for the case of the roll velocity potential, though the center of the rotation of the single hull section locates at the center of a hull, that of the twin hull section locates at the center of the twin hulls. Then we must add a heave velocity potential oscillating opposite direction between the left and right hull with amplitude $P/2$ to the roll velocity potential. Namely, the velocity potential of the rolling twin hull section ψ_3 is expressed as follows.

$$\begin{aligned}\psi_3(y,z)&=\varphi_3(y,z)+C_3\{\varphi_2(y,z)-\overline{\varphi}_2(y,z)\}+D_3\{\varphi_1(y,z)-\overline{\varphi}_1(y,z)\}\\&\quad-\frac{P}{2}[\varphi_2(y,z)+C_2'\{\varphi_2(y,z)-\overline{\varphi}_2(y,z)\}+D_2'\{\varphi_1(y,z)-\overline{\varphi}_1(y,z)\}]\end{aligned} \tag{46}$$

The interaction coefficients C_2' and D_2' are obtained as the values of C_2 and D_2 replacing $(-1)^j$ with -1 in (45).

The velocity potential on the right hull section can be calculated by that on the left hull section using the symmetry or anti-symmetry of the entire flow.

$$\psi_j(Y,Z)=(-1)^j\psi_j(-Y,Z) \tag{47}$$

Pressure on the hull sections can be calculated by a following equation.

$$P_j(Y,Z)\mathrm{e}^{i\omega t}=\rho i\omega\xi_j\frac{\partial\psi_j(Y,Z)\mathrm{e}^{i\omega t}}{\partial t}=-\rho\omega^2\xi_j\psi_j(Y,Z)\mathrm{e}^{i\omega t} \tag{48}$$

Added mass A_{ij}^l and damping coefficient B_{ij}^l of the left hull section to the coordinate system $o-y,z$ and those of the right hull section A_{ij}^r and B_{ij}^r are calculated by following equations.

$$A_{ij}^l=-\mathrm{Re.}\left[\frac{F_{ij}^l}{\omega^2\xi_j}\right],\ \ B_{ij}^l=\mathrm{Im.}\left[\frac{F_{ij}^l}{\omega\xi_j}\right],\ \ A_{ij}^r=(-1)^{i+j}A_{ij}^l\,,\ \ B_{ij}^r=(-1)^{i+j}B_{ij}^l \tag{49}$$

Total added mass A_{ij} and damping coefficient B_{ij} of the coordinate system $O-Y,Z$ for the twin hull section is calculated by followig equations.

$$A_{ij}=-\mathrm{Re.}\left[\frac{F_{ij}}{\omega^2\xi_j}\right],\ \ B_{ij}=\mathrm{Im.}\left[\frac{F_{ij}}{\omega\xi_j}\right] \tag{50}$$

Where, F_{ij}^{l} is a force or a moment calculating by pressure integration as

$$F_{ij}^{l} = \int_{C_L} n_i P_j(y,z) ds \tag{51}$$

and

$$F_{ij} = F_{ij}^{L} + F_{ij}^{R} = \int_{C_L} N_i P_j(Y,Z) ds + \int_{C_R} N_i P_j(Y,Z) ds \tag{52}$$

When we introduce the wave interaction between hulls to the calculation of hydrodynamic coefficients, even if an individual hull section is symmetric, the coupling terms of (49) of the individual section as A_{23}^{l} still exist. But these terms are canceled out to the entire twin hull section. The hydrodynamic coefficients of the individual section are needless for the ship motion computation but require the calculation of sea loads acting on the cross deck structure.

2.3 Diffraction problems for a twin hull section

In this section, we describe the computing method of scattering pressure and wave exciting forces and moments by the Bessho's relation (12) using the aforementioned radiation potentials. We rewrite (12) to the twin hull section as

$$\psi_s^{\pm} = \frac{\bar{H}_k^{\mp}(\psi_j - \bar{\psi}_j) - \bar{H}_j^{\mp}(\psi_k - \bar{\psi}_k)}{i(\bar{H}_j^{\mp}\bar{H}_k^{\pm} - \bar{H}_j^{\pm}\bar{H}_k^{\mp})} \tag{53}$$

Where, $\psi_s^{\pm}$ is a scattering potential. $H_j^{\pm}$ is the Kochin function of the twin hull section for j-mode motion and the double sign $\pm$ denotes the direction of outgoing wave along Y axis. $\bar{H}_j^{\pm}$ is the complex conjugate of $H_j^{\pm}$.

(53) requires the Kochin function $H_j^{\pm}$. The Kochin function of the left hull section $H_j^{L\pm}$ is defined to the coordinate system $O-Y,Z$ as

$$H_j^{L\pm} = \int_{C_R} \left(N_j - \psi_j \frac{\partial}{\partial N} \right) \mathrm{e}^{-K(Z \mp iY)} ds, \ (j = 1, 2) \tag{54}$$

Substituting $Y = y - P/2$ and $Z = z$ into (54)

$$H_j^{L\pm} = \mathrm{e}^{\mp iKP/2} \int_{C_R} \left(n_j - \psi_j \frac{\partial}{\partial n} \right) \mathrm{e}^{-K(z \mp iy)} ds \tag{55}$$

By using (10), (11) and (55), $y = Y + P/2$, the Kochin function of the left hull section can be written as

$$H_j^{L\pm} = \mathrm{e}^{\mp iKP/2} (\Sigma_j \pm iM_j) \tag{56}$$

In the same way, the Kochin function of the right hull section can be written as

$$H_j^{R\pm} = (-1)^j \mathrm{e}^{\pm iKP/2} (\Sigma_j \mp iM_j) \tag{57}$$

Therefore, the entire Kochin function of the twin hull section $H_j^{\pm}$ can be written as

$$H_j^{\pm} = H_j^{L\pm} + H_j^{R\pm} = \begin{cases} \mp 2i\{\Sigma_j \sin(KP/2) + M_j \cos(KP/2)\}, & j=1 \\ 2\{\Sigma_j \cos(KP/2) - M_j \sin(KP/2)\}, & j=2 \end{cases} \tag{58}$$

Substituting (21), (47) and (58) for j and $k(\neq j)$ mode into (53), we obtain the scattering potential $\psi_S^{\pm}$ and subtracting the following incident wave potential from $\psi_S^{\pm}$,

$$\Psi_0^{\pm}(Y,Z)\mathrm{e}^{i\omega t} = i\frac{g\zeta_w}{\omega}\psi_0^{\pm}(Y,Z)\mathrm{e}^{i\omega t}, \quad \psi_0^{\pm}(Y,Z) = \mathrm{e}^{-K(Z\pm iY)} \tag{59}$$

we obtain the diffraction potential $\psi_4^{\pm}$ as

$$\psi_4^{\pm} = \psi_S^{\pm} - \psi_0^{\pm} \tag{60}$$

Scattering pressure $P_S^{\pm}$ is calculated as follows

$$P_S^{\pm}(Y,Z)\mathrm{e}^{i\omega t} = \rho\frac{ig\zeta_w}{\omega}\frac{\partial\psi_S^{\pm}(Y,Z)\mathrm{e}^{i\omega t}}{\partial t} = -\rho g\zeta_w\psi_S^{\pm}(Y,Z)\mathrm{e}^{i\omega t} \tag{61}$$

Wave exciting forces and moment $E_j^{\pm}$ for j-mode direction are computed by the pressure integral as

$$E_j^{\pm} = E_j^{L\pm} + E_j^{R\pm} = \int_{C_L} N_j P_S^{\pm}(Y,Z)ds + \int_{C_R} N_j P_S^{\pm}(Y,Z)ds \tag{62}$$

and by the Haskind relation

$$E_j^{\pm} = -\rho g\zeta_w H_j^{\mp} \tag{63}$$

The wave exciting forces of the individual section are needless for the ship motion computations but require that of the sea loads acting on the cross deck structure. The calculation method of $E_j^{L\pm}$ and $E_j^{R\pm}$ by using the Haskind relation has already proposed by Ohkusu [6]. We notice the wave exciting forces $E_j^{L,R\pm}$ of (62) are defined to the coordinate system $O-Y,Z$.

3 Numerical examples and discussions

We computed the pressure distributions and hydrodynamic forces and moments for the Lewis form section with the sectional area ratio σ=0.941 and the half beam draught ratio H_0=1.0. Radiation pressures of a single hull section were computed by the program of the singularity distribution method developed by Bedel et al. [13]. The unit amplitude velocity potentials can be reproduced from the radiation pressures as following.

$$\varphi_j = (p_a - ip_v)/K \tag{64}$$

Where; p_a is the pressure in acceleration phase and p_v is that in velocity phase. The distance between hulls $P/T = 4$.

Fig.3 shows the pressure distribution in acceleration phase P_a and Fig.4 shows that in velocity phase P_v by rectangles heaving at KT=0.1(T: draught). In Fig.3 and 4, the

upper side figure shows the pressure distribution of the single hull section, the lower left figure shows that of the left hull section of the twin hull computing with the local wave terms of (39) ~ (41) and the lower right without the local wave terms. All of the pressure figures are upside down to phisical space, because of z axis positive downward.

The pressure distribution has been non-dimentinalized by dividing the static pressure at the depth of twice amplitude of heaving. And each rectangle shows the working direction and amplitude of the pressures. These figures show the influences of the local wave terms to radiation pressures are small even in the case of low frequency.

Fig.5 shows the scattering pressure distribututions of the cos-component and Fig.6

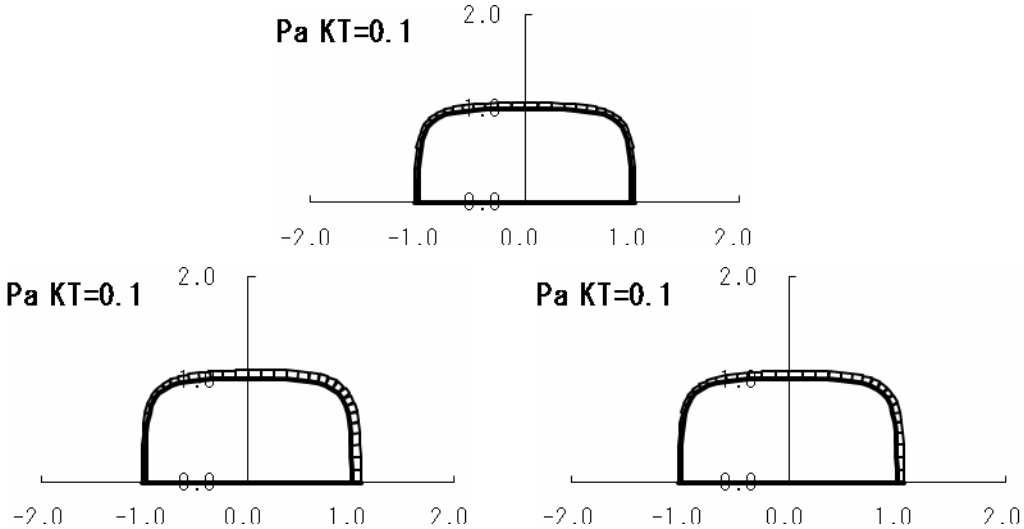

Fig. 3 Pressure distributions in accelaration phase by heaving motion
(The upper: mono-hull, the lower-left: with local wave, the lower-right: without local wave)

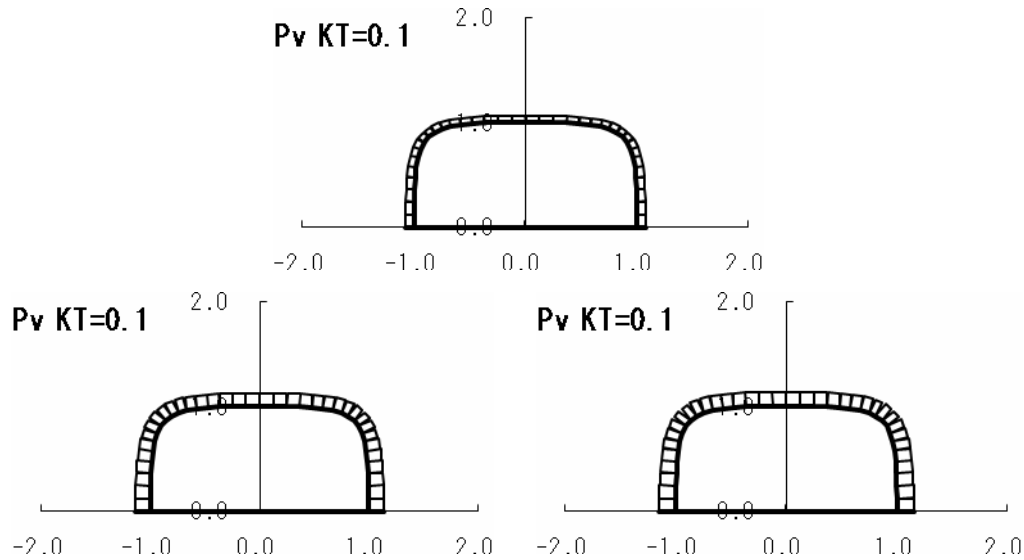

Fig. 4 Pressure distributions in velocity phase by heaving motion
(The upper: mono-hull, the lower-left: with local wave, the lower-right: without local wave)

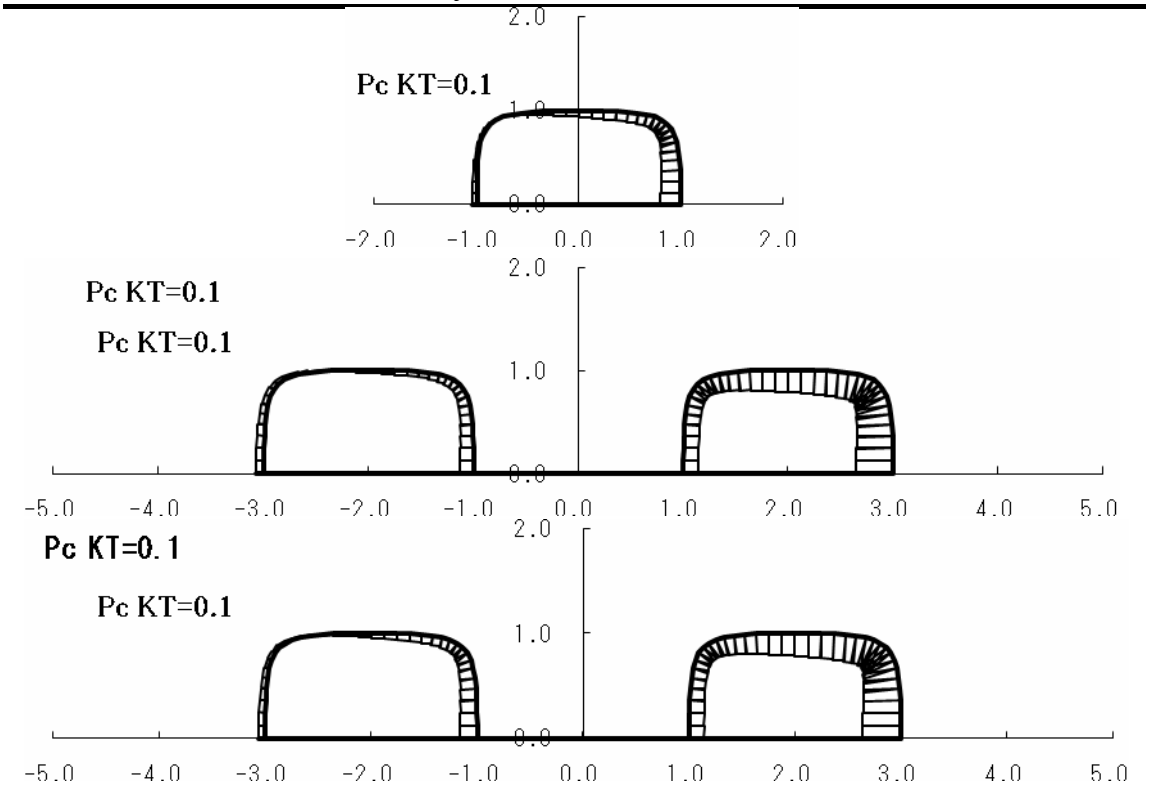

Fig. 5 Pressure distributions of the Cos-component by wave scattering

(The upper: single hull, the middle: twin hull with local wave, the lower: without local wave)

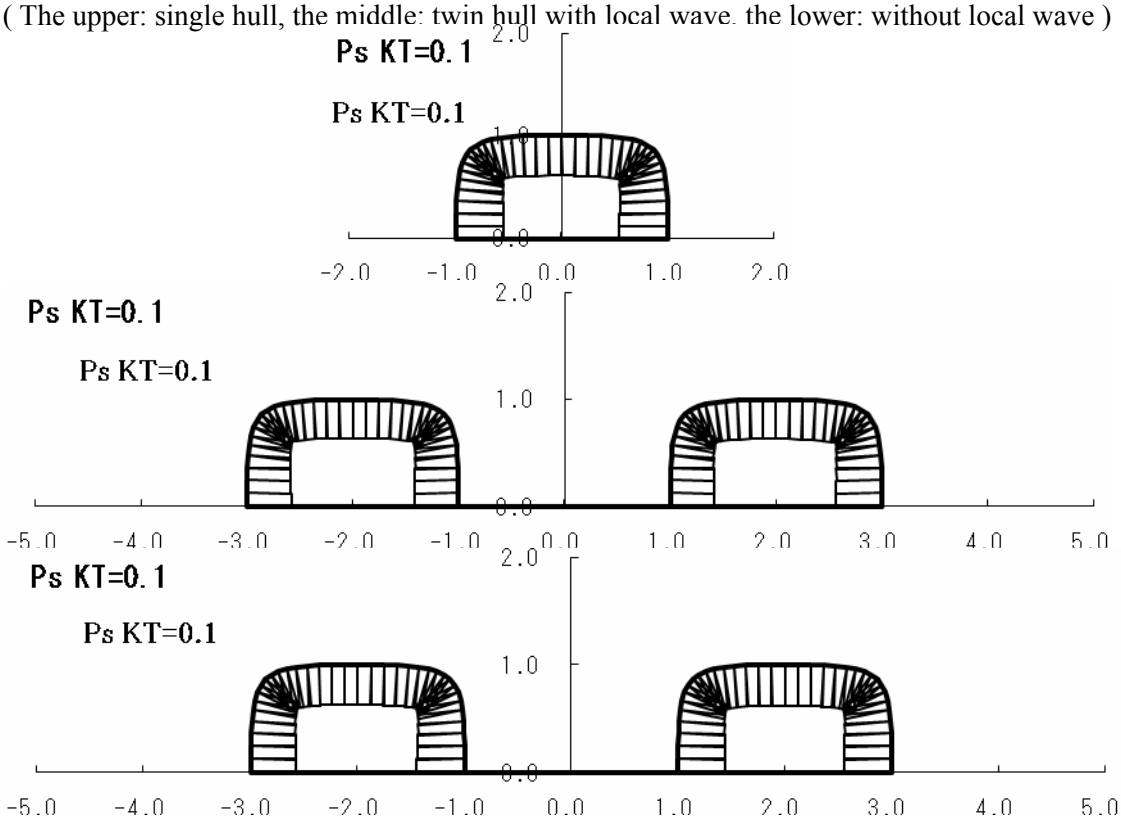

Fig. 6 Pressure distributions of the Sin-component by wave scattering

(The upper: single hull, the middle: twin hull with local wave, the lower: without local wave)

shows that of the sin-components. Fig.5 and 6, the upper side figure shows the pressure distribution of the single hull section, the middle figure shows that of the twin hull section computing with the local wave terms of (39) ~ (41) and the lower figure without

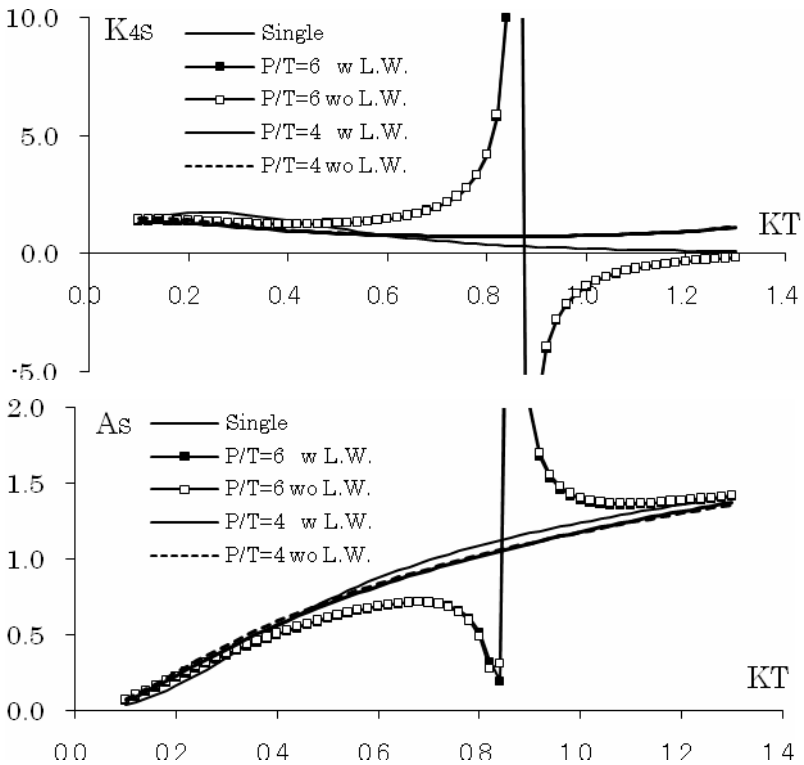

Fig.7 Added mass coefficient and wave amplitude ratio for swaying catamaran

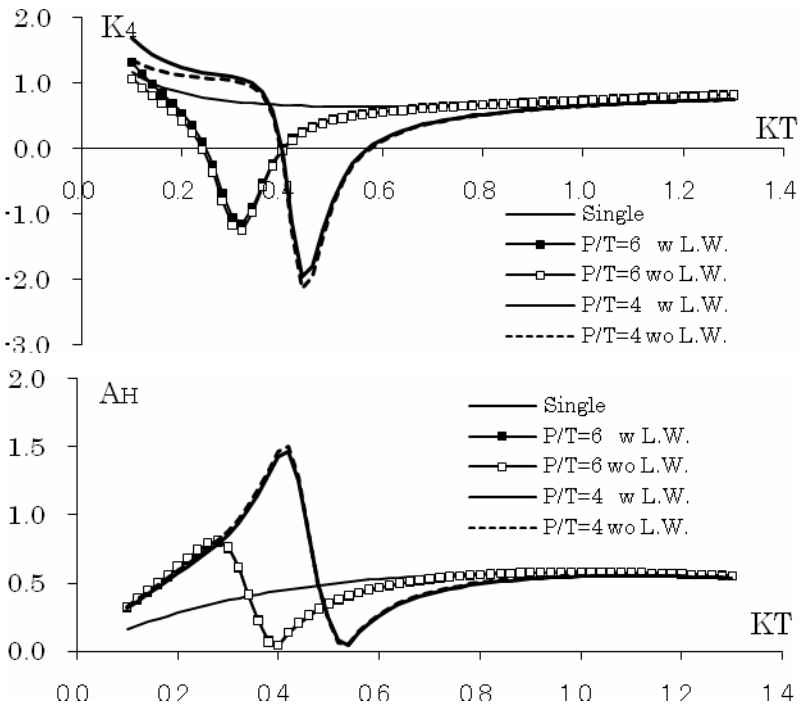

Fig.8 Added mass coefficient and wave amplitude ratio for heaving catamaran

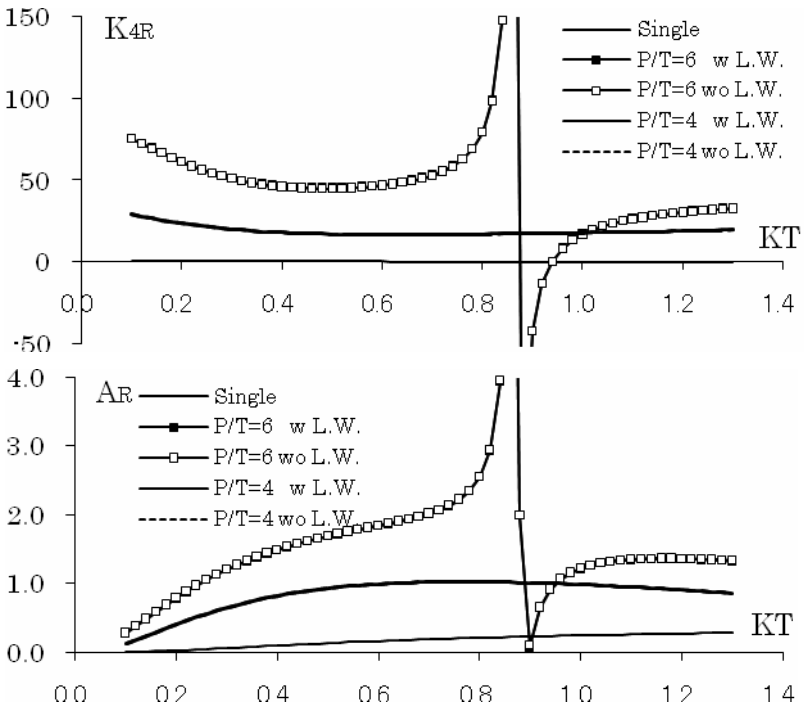

Fig.9 Added moment of inertia coefficient and wave amplitude ratio for rolling catamaran

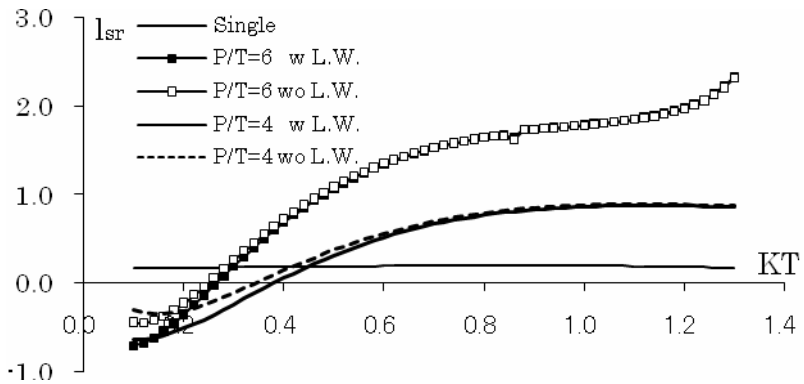

Fig.10 Levers of in phase coupling moment for swaying catamaran

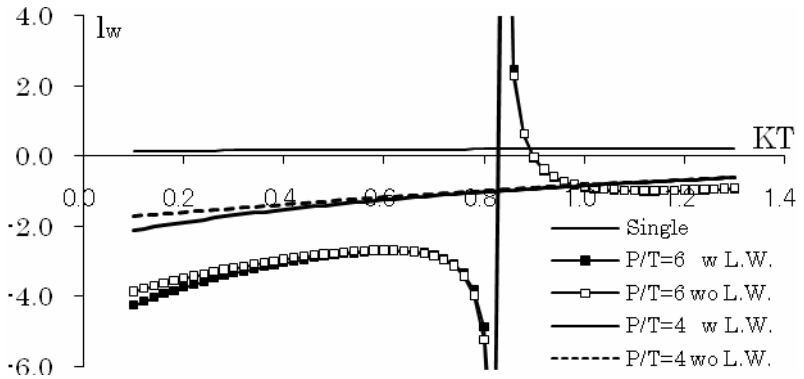

Fig.11 Levers of out of phase coupling moment for swaying catamaran

local wave terms. These figures show the influences of the local wave terms to scattering pressures are small even in low frequency.

Fig.7 ~ 11 shows the added mass and damping coefficients non-dimensionalized as same manners of the reference [13].

$$K_{4S} = \frac{A_{11}}{\pi T^2}, \quad K_4 = \frac{A_{22}}{\pi / 4B^2 C_0}, \quad K_{4R} = \frac{A_{33}}{\pi / 4T^4}, \quad l_{sr} = \frac{A_{31}}{A_{11}T}, \quad l_w = \frac{B_{31}}{B_{11}T} \tag{65}$$

Where, C_0 is the heaving added mass for the infinite frequency. A_S, A_H and A_R is the wave amplitude ratio of sway, heave and roll motion. In legends of these figures, "Single" shows the results of the single hull section and the other shows that of twin hul section. And also, "w L.W." or "wo L.W." shows the calculations with or without the influence of the local wave terms. There are some differences between the results of "w L.W." and "wo L.W." for all of the motions in low frequency region.

Fig.12 ~ 14 show the wave exciting forces and moment by the pressure integral of (61). The results were non-dimensionalized as same as the reference [4] as follows.

$$E_1 = \frac{E_1^-}{\rho g \zeta_w T}, \quad E_2 = \frac{E_2^-}{\rho g \zeta_w T}, \quad E_3 = \frac{E_3^-}{\rho g \zeta_w T^2} \tag{66}$$

In the Fig.8, the frequency at which the heaving wave amplitude ratio goes to zero is called a wave free frequency and at that the wave exciting force for heave direction also goes to zero as shown in Fig.13. The wave free frequency of twin hull section arises by

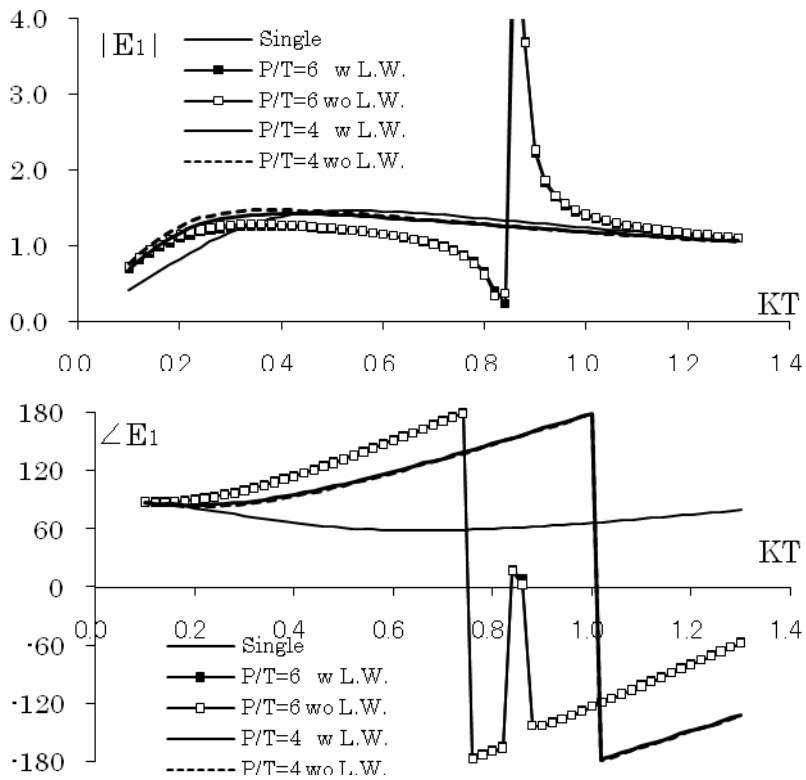

Fig.12 Wave exciting sway force and its phase lag for catamaran

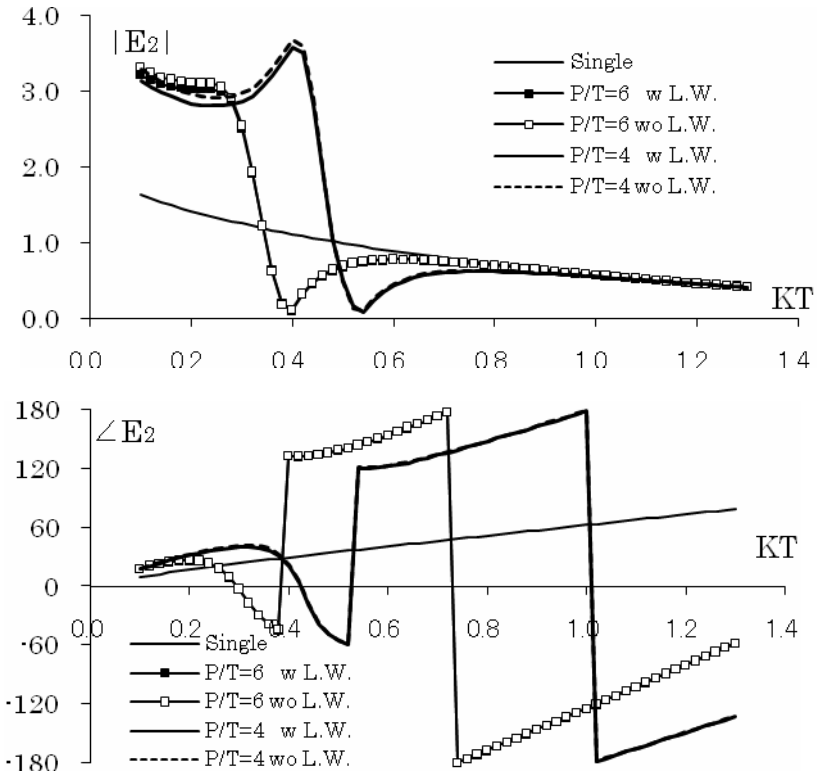

Fig.13 Wave exciting heave force and its phase lag for catamaran

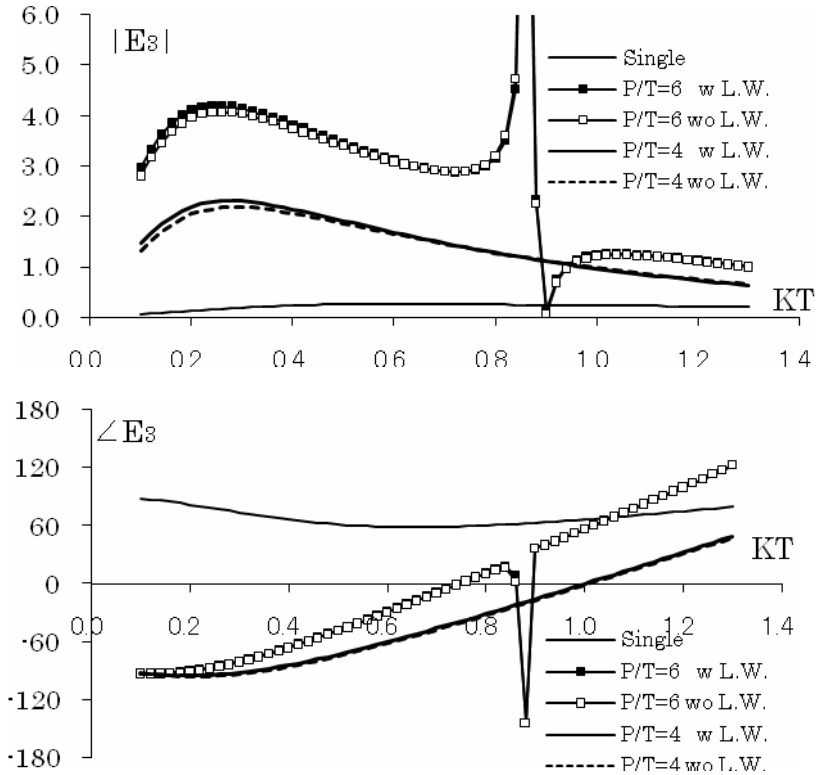

Fig.14 Wave exciting rollong moment and its phase lag for catamaran

the wave interaction between hulls. As described in the reference [6], the pressures on the hulls do not become to zero but the component of the pressure to the heave direction cancelled out between hulls. Fig.12 ~ 14 shows the influence of the local wave come to remarkable with the shorter distance between hulls and lower frequency.

Computed pressures could not be confirmed with experiments but the forces and moments by pressure integral agreed with computations of Takezawa et al. [5].

The methods used in this chapter are called the far field approximation [10] from the view point of the slender body theory. When we write the ship length is L, the breadth of hull section is B, we assume B much smaller than L, namely $B/L=\varepsilon<<1$, the far field approximation requires the assumption that the distance between hulls P is $P/L=O(1)$. On the other hand, the near field approximation assumes $P/L=O(\varepsilon)$ and defines the strict 2D two bodies' problem. The near field approximation is the strict 2D solution, but in the view point of the approximation of the 3D problem, that is still approximated solution as same as the far field approximation.

4 Conclusions

We have described the calculation method of the radiation/diffraction pressures with wave interaction of a twin hull section from the results of a single hull section. The method can calculate not only forces and moments but also pressures on the hulls. In this paper, all of the results for the twin hull section have been obtained from the output of the pressure distributions for the single hull section without solving any boundary value problems. The calculation for the twin hull section requires only 1 % increase of computing time for a single hull section [5].

Present method provides the pressure on a catamaran stopping in beam seas which is utilized for the FEM structural analysis and ship motion prediction in waves. On applying to wave power boats, accurate thrust prediction requires accurate ship motion prediction. By using the hydrodynamic forces including the wave interaction between hulls, we can predict the accurate motions of a catamaran in beam seas [7].

Now, Prof. Terao the developer of "Mermaid II" is going to apply these computater codes to the design of a next wave power boat.

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